Report (5) 11th Jan 2013	**Beam Position Monitor Calculations**	P. Forck M. Almalki
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Web-Link:	http://www-bd.gsi.de/dokuwiki/doku.php?id=projects:bpm-linac	

This short report presents a description of the BPM position sensitivity and mapping calculations [1].

➢ The Image Current Density

Considering a pencil centered beam I_{beam} in a circular beam pipe, the wall current density j_{im} is given as a function of the azimuthal angle $\emptyset$ by :

$$j_{im} = \frac{I_{beam}}{2\pi a}\left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos(\emptyset - \theta)}\right) \quad (1)$$

Where a is the beam pipe of radius = 25 mm and r, θ are the beam position. Considering an azimuth $\theta = 0$, the image current density generated by a 'pencil' beam at different displacements r has been calculated as depicted in figure 1. "Note : we consider ($I_{beam}/2\pi a$) as a constant".

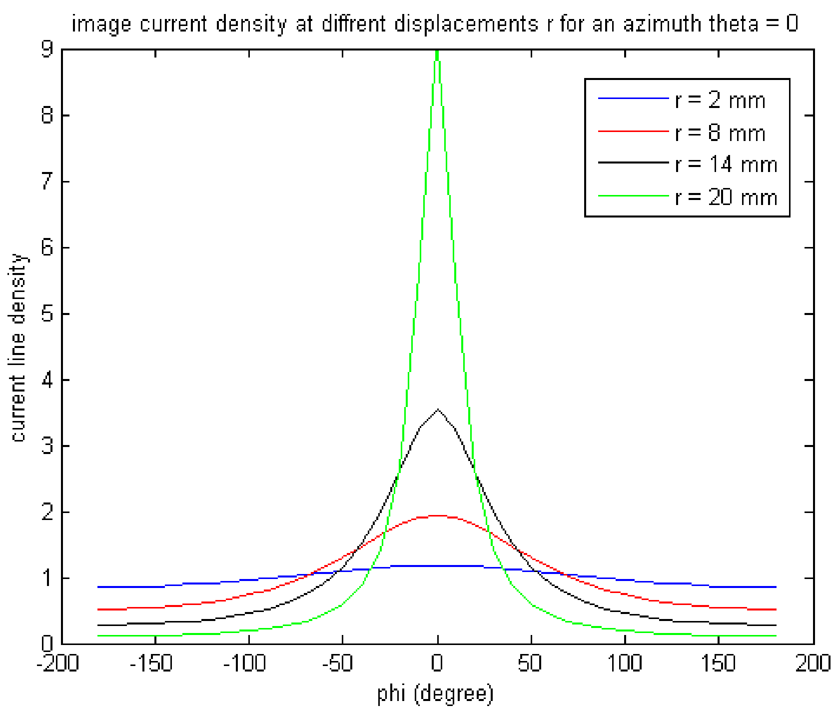


Figure 1,

➢ Transverse Position Calculations

The transverse beam position in horizontal directions is determined by calculating the induced voltage on two opposite electrodes using the difference over sum methode and similarily for he vertical directions. The buttons covering an angle α and the image current j_{im} is recorded as given by:

$$I_{im} = a \int_{-\alpha/2}^{+\alpha/2} j_{im}\,(\emptyset) d\emptyset \tag{2}$$

The horizontal displacement has been calculated at different values of the azimuthal orientation θ $(0^{\circ}, 20^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 70^{\circ})$ and $\alpha = 84^{\circ}$ showon in figure 2. Using eq (2), the image current I_{im} has been calculated for the two oposite electrodes which is the integration of the wall current density j_{im}.

an example, the plane of $\emptyset = 20^{\circ}$:

$$I_{electrod1} = \frac{I_{beam}}{2\pi} \int_{-84/2}^{84/2} \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos\,(\emptyset - 20)} \right) d\emptyset$$

$$I_{electrod2} = \frac{I_{beam}}{2\pi} \int_{-\frac{84}{2}+180}^{\frac{84}{2}+180} \left(\frac{a^2 - r^2}{a^2 + r^2 - 2ar \cdot \cos\,(\emptyset - 20)} \right) d\emptyset$$

Where r = $r = x/\cos\,(20)$, x takes the value from -25 to 25.

$$H = \frac{\Delta I_{im}}{\Sigma I_{im}} = \frac{I_{electrod1} - I_{electrod2}}{I_{electrod1} + I_{electrod2}} \tag{3}$$

From plots in figure 2, the position sensitivity (S_x) was calculated from the slope, $S_x = 7\ \%/mm$. S_y is the position sensitivity for vertical displacement.

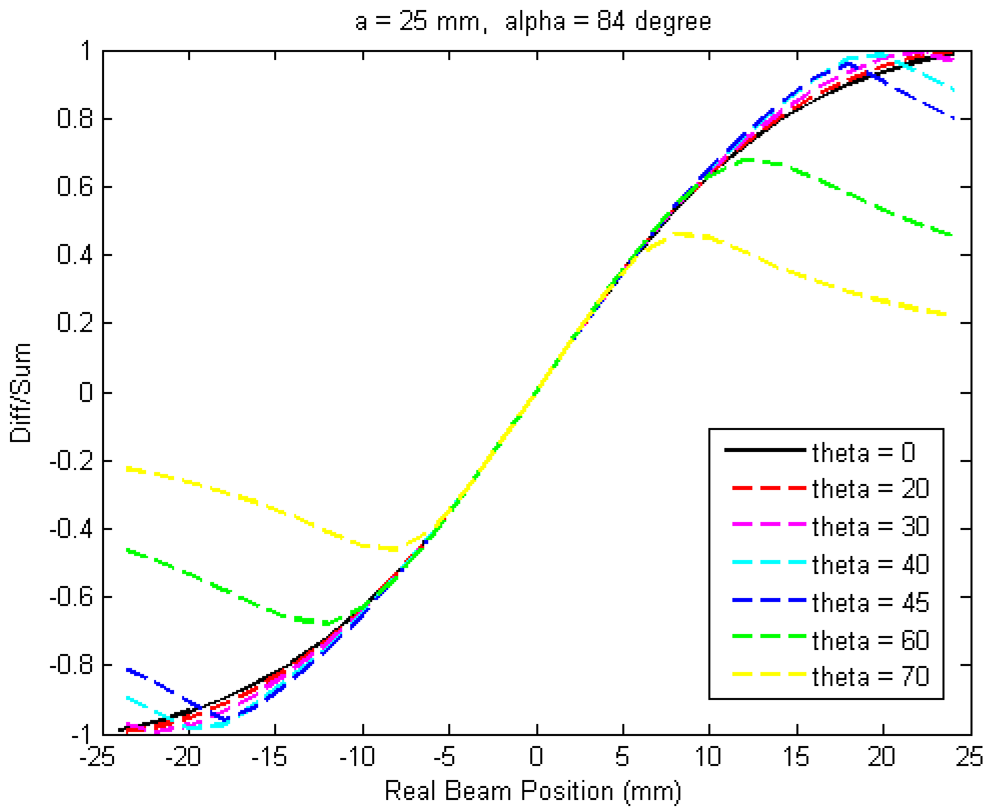


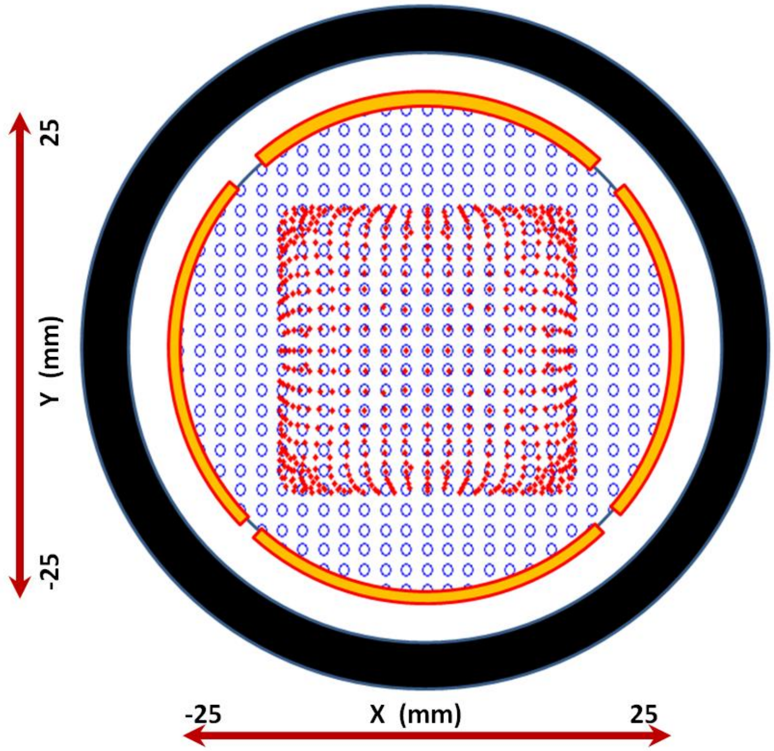
Figure 2, amplitude-ratio response for the BPM with a 50 mm diameter aperture and 84°electrode.

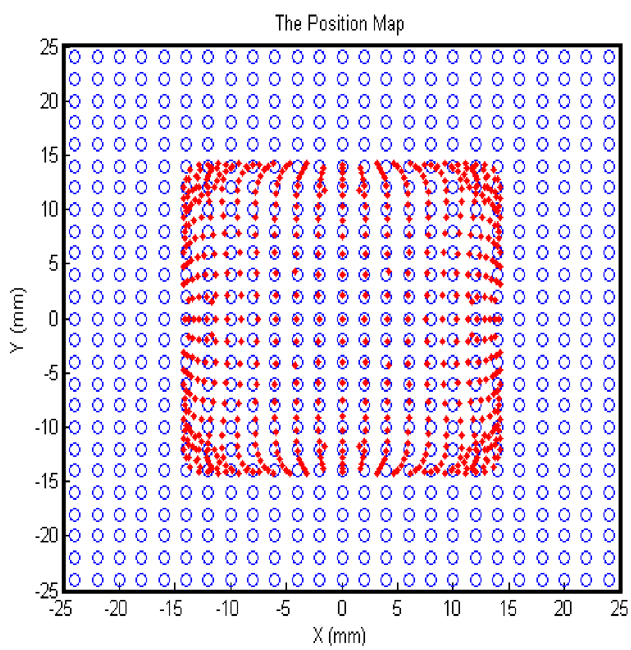
➢ The Beam Position Map

Using the result from the previous section, the position map is calculated where the open circles represent the real beam position and the dots are the results of the 1/S ·ΔU/ΣU algorithm with S = 7 % / mm for the central part.

$$Real\ beam\ Position = \frac{1}{S_x} \cdot \frac{\Delta I_{im}}{\Sigma I_{im}} = k_x \cdot \frac{I_{electrod1} - I_{electrod2}}{I_{electrod1} + I_{electrod2}}$$

The real position was calculated in a step of 2 mm for a range of -24 mm to 24 mm as it shown in the next page.





[1] The method is described in the "Lecture Notes on Beam Instrumentation and Diagnostics, P. Forck, JAUS March 2012".

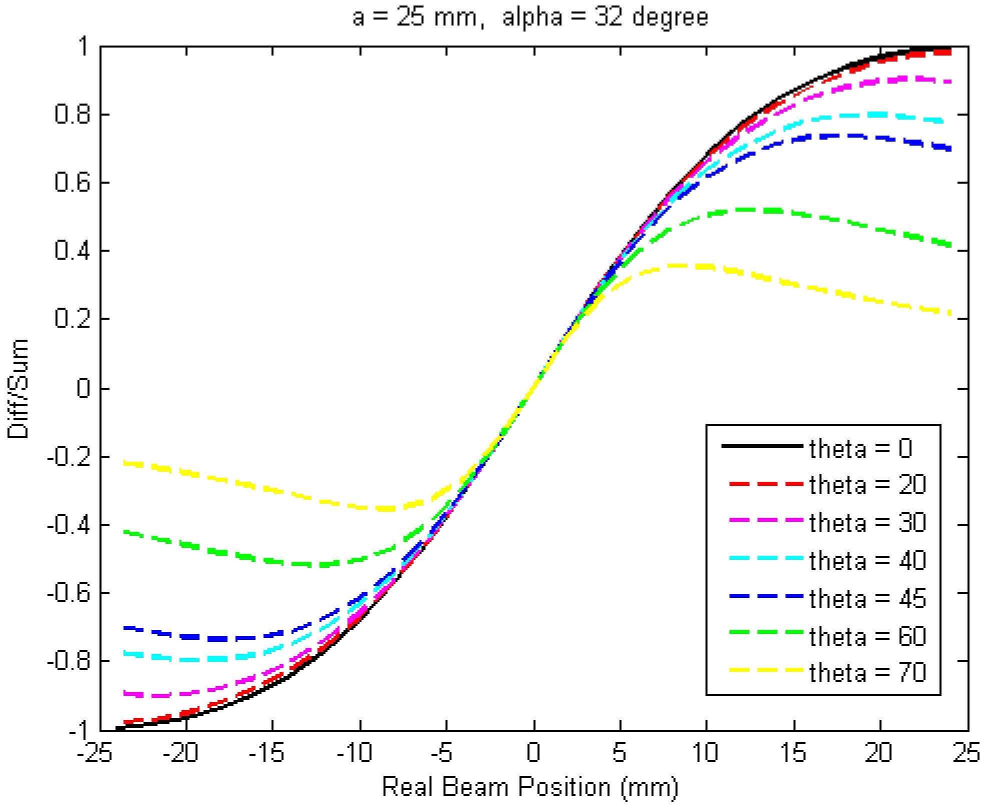


Figure 3, amplitude-ratio response for the BPM with a 50 mm diameter aperture and 32°electrode.