The attenuation of external magnetic fields by superconducting simple and coaxial cylinders are given. It is shown that inserting a superconducting rod into a simple superconducting cylinder of radius r reduces the attenuation of fields normal to the z-axis from exp (− 1.84 z/r) to exp (− 1.0 z/r) in the narrow gap limit.

Magnetic shielding by superconducting simple and coaxial cylinders: a comparison

K. Grohmann and D. Hechtfischer

In cryogenic experiments it is frequently necessary to screen parts of the apparatus against magnetic fields. The reason may be the need for an ultra-low field region[1] or the necessity to protect a sensitive part (eg a SQUID) against environmental magnetic noise. In addition, with cryo current comparators[2-4] a metrological application of superconducting shielding has come into use. A frequently used shielding element is the circular cylinder, either as a shield by itself or in connection with other shields in order to protect feedthroughs against magnetic leakage. It is the purpose of this paper to present the attenuation exhibited by such cylinders for different conditions of magnetic field and geometric parameters. Especially, it will be shown that the situation changes drastically if the bore of a cylinder is partly filled with other superconducting material, thus forming coaxial cylinders. This will happen, for example, if an insulated superconducting wire or an adjustment rod, made from superconducting material, passes the bore. As will be demonstrated, the shielding of transverse fields by such coaxial cylinders is normally worse than the shielding provided by simple cylinders, in spite of the reduced cross-section.

Theory

The attenuation of magnetic fields by superconducting cylinders can be found by solving the magnetic boundary problem. The theoretical fundamentals and performance of the calculations have been described in connection with cryo current comparators.[5] Therefore, it is sufficient here to outline the method and to give the results of the calculations. In Fig. 1 the geometry discussed is shown. The coaxial cylinders degenerate to a simple cylinder if the radius r_i of the inner superconducting rod is supposed to be zero.

Using the Maxwell-London Theory in its ideal diamagnetic limit[6] and considering stationary cases, we can derive an expression for the magnetic field outside a superconductor in terms of scalar potential V.

$$\mathbf{B} = -\nabla V \tag{1}$$

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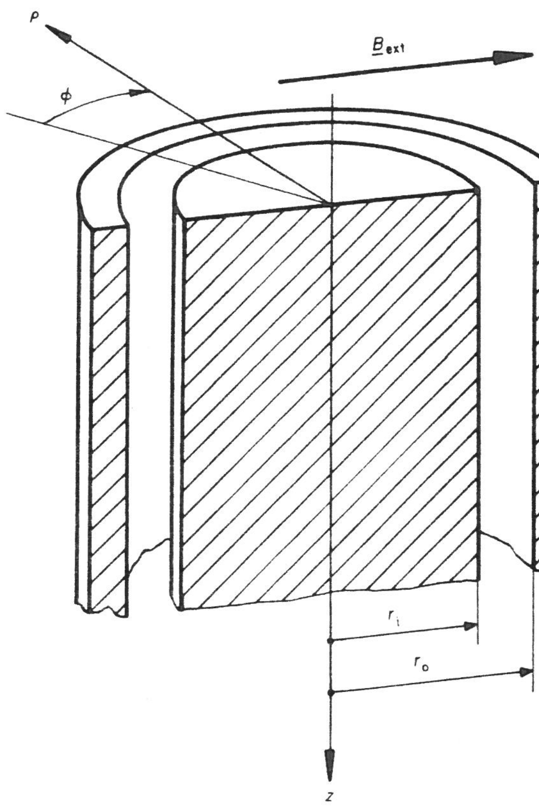

Fig. 1 Coaxial cylinders

which obeys the Laplace equation

$$\Delta^2 V = 0. \tag{2}$$

The diamagnetism of the superconducting shields is described by

$$\frac{\partial V}{\partial n} = 0 \tag{3}$$

on all shield surfaces, where $\partial/\partial n$ denotes the derivative with respect to the normal of the surfaces. The solution of

Table 1. Decrease of the magnetic potential in superconducting cylinders

	Simple cylinder	Coaxial cylinders ($r_0/r_i = 1.1$)
Transverse field	$V_{11}\exp(-1.84\,z/r_0) + V_{12}\exp(-5.33\,z/r_0) + \ldots$	$V_{11}\exp(-1.05\,z/r_0) + V_{12}\exp(-34.5\,z/r_0) + \ldots$
	$+ V_{21}\exp(-3.05\,z/r_0) + V_{22}\exp(-6.71\,z/r_0) + \ldots$	$+ V_{21}\exp(-2.10\,z/r_0) + V_{22}\exp(-34.6\,z/r_0) + \ldots$
	$+ \ldots$	
Longitudinal field	$V_{01}\exp(-3.83\,z/r_0) + V_{02}\exp(-7.02\,z/r_0) + \ldots$	$V_{01}\exp(-34.6\,z/r_0) + V_{02}\exp(-69.1\,z/r_0) + \ldots$

the Laplace equation for our cylindrical problem is given by a twofold infinite series:

$$V(\rho, \phi, z) = \sum_{n=0}^{\infty} \sum_{\text{all}\,k} V_{nk}(k\rho)$$

$$\sin(n\phi + \nu_n)\exp(-kz) \tag{4}$$

(ρ, ϕ, z are cylindrical coordinates, ν_n is a phase angle). The index n characterizes the harmonics of a Fourier series of potential $V(\rho, \phi, z)$ with respect to the angle ϕ, and thus different field configurations may be distinguished. For example, an external homogeneous field normal to the cylinder axis or a dipolar field centred at $\rho = 0$ will generate only the first 'mode' $n = 1$ of the potential, a quadrupolar field generates only the second mode $n = 2$, etc. A longitudinal field without angular dependence is described by the mode $n = 0$.

As we are interested in the attenuation of the applied field with increasing z, we have to determine the values which k can assume by solving the corresponding eigenvalue problem. It is found that each mode n has its own set of eigenvalues $k_1^n, k_2^n, \ldots, k_m^n \ldots$, where the subscript m gives the number of extrema the radial field component has between $\rho = r_0$ and $\rho = r_i$. Therefore fields of different symmetry, that is different n, will be attenuated in an individual manner. In Table 1 the most important of these solutions is given for the simple and coaxial cylinder arrangement with ratio $r_0/r_i = 1.1$. Since the z is related exponentially to the potential, these solutions fully describe the relation between z and the corresponding magnetic fields, because $\mathbf{B} = -\nabla V$. Defining an attenuation factor A_{nm} for each component, (n, m) of the potential series according to

$$A_{nm} = \frac{V(z, n, m)}{V(z = 0, n, m)} = \exp(-k_m^n z) \tag{5}$$

we get for transverse fields the lowest attenuation for $n = 1$ and $m = 1$:

simple cylinder: $A_{11} = \exp(-1.84\,z/r_0)$ (6a)

coaxial cylinders: $A_{11} \to \exp(-1.0\,z/r\ )$ for $r_0/r_i \to 1$ (6b)

The attenuation by the coaxial cylinders depends on the ratio of the radii.[5] For $r_0/r_i \leqslant 1.5$ the following approximation holds:

$$A_{11} \cong \exp\left\{-\left(\frac{2}{1 + r_i/r_0}\right)\frac{z}{r_0}\right\} \tag{6c}$$

For $r_0/r_i > 10$ the attenuation can be approximated by (6c) for the simple cylinder.

The results show that the attenuation of transverse fields by coaxial cylinders is less than that by simple cylinders. For longitudinal fields the opposite is true:

simple cylinder: $A_{01} = \exp(-3.83\,z/r_0)$ (7a)

coaxial cylinders: $A_{01} \to \exp(-\infty\,z/r_0)$ for $r_0/r_i \to 1$ (7b)

For $r_0/r_i \leqslant 1.5$ A_{01} can be approximated by

$$A_{01} \cong \exp\left\{-\left(\frac{\pi}{1 - r_i/r_0}\right)\frac{z}{r_0}\right\} \tag{7c}$$

Again for $r_0/r_i > 10$ the result for the simple cylinder approximately holds. If the length of the cylinders is large compared with the radius r_0, then the attenuation factors are so small compared to A_{11}, that the corresponding potentials or fields can be neglected. This means that at larger values of z the largest remaining term, if it still can be measured, will be the dipolar one. The cylinders act like a filter, which 'selects' the dipolar component of magnetic fields of arbitrary symmetry.

Experiments

The attenuation of longitudinal fields is much stronger than that of transverse fields. Therefore, any transverse leakage field, which is very difficult to avoid in an experiment, will normally cover signals from longitudinal fields. For this reason we will confine our work to attenuation measurements of transverse fields, which represents all the practical, relevant cases.

The experimental set-up is shown in Fig. 2. The detection coil of a flux transformer coupled to a SQUID is shielded against external fields by means of a lead box. The only way in which external fields can enter the detection area is through the chimney on top of the box. The outer cylinder has an inner diameter of $2r_0 = 15.6$ mm and is made of lead ($T_c = 7.2$ K). The insulated core has a diameter of $2r_i = 12.0$ mm and is made of tin ($T_c = 3.7$ K). This arrangement facilitates a 'switch' from a simple superconducting cylinder to coaxial cylinders by lowering the temperature of the helium bath from 4.2 K to a value below the transition temperature of the tin core.

In Fig. 3 the registered SQUID signal, resulting from a transverse external field is shown for various cylinder lengths l. In addition the theoretical results are shown (slopes calculated, absolute values adapted to the experimental values). The measurements agree with the calculated effects of the cylinder length. Within the accuracy limits the simple cylinder (sc) the measured attenuation factor $A^{sc} = \exp(-1.84\ l/r_0)$ and for the coaxial cylinders (cc) with the ratio of radii $r_0/r_i = 1.3$ $A^{cc} = \exp(-1.13\ l/r_0)$. Due to this lower attenuation, the signals always increase after switching to the coaxial cylinder state.

Discussion

The results show that one has to be careful with 'common sense' estimations of the effectivity of superconducting shielding systems. Such an estimation could predict the coaxial cylindrical gap to be superior because of the smaller cross-section. Of course the cross-section will influence the detected signal at a given length, because it determines the amount of flux which couples with the detector coil. In the small gap limit, the flux decreases linearly with the width $d = r_0 - r_i$ of the gap.[5] For comparison with the measured values ($d = 1.8$ mm), the calculated length dependence of the signal for a gap width of 0.1 mm ($r_0/r_i \cong 1.01$) is shown by the dashed line in Fig. 3. It has the lower inclination $\exp(-1.0\ l/r)$ and starts at $l = 0$ with a value which is reduced by the quotient of the respective gap widths. Now for a shielding length smaller than about two diameters, the coaxial shield system is superior to the simple cylinder, but with the exception of such short narrow gaps, for a given diameter, the simple cylinder will be superior.

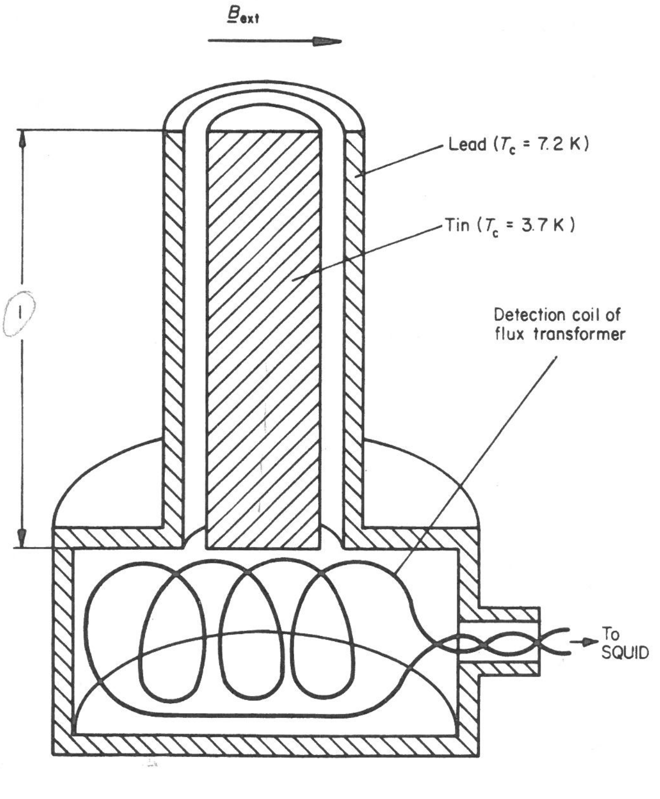

Fig. 2 Measurement of field attenuation by superconducting cylinders. At $T = 3.7$ K the arrangement switches from a simple superconducting cylinder to coaxial cylinders.

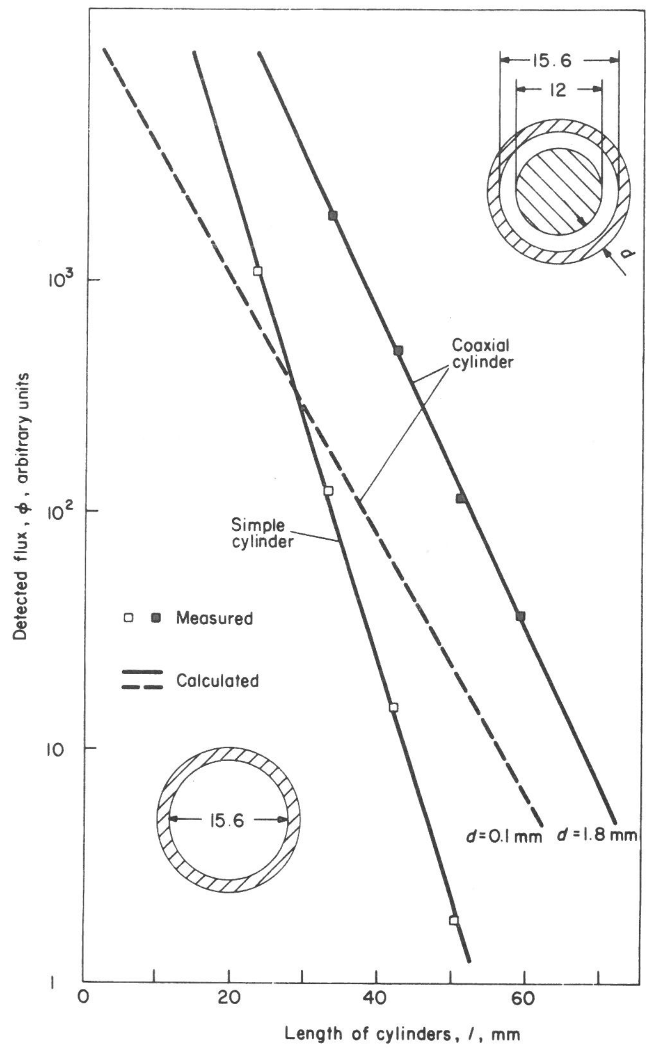

Fig. 3 Detected flux as a function of the cylinder length for an unchanged external transverse field in the simple and coaxial cylinder state. Except for extremely narrow gaps at short lengths the simple cylinder is superior to the coaxial cylinders.

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Cryostats

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Fig. 2 [illegible]

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