Measurement of longitudinal Parameters

The longitudinal dynamics is described by the longitudinal emittance as given by:

- Spread of the bunches ***l*** in time, length ***or*** rf-phase.
- Momentum spread $\boldsymbol{\delta = \Delta p/p}$, or energy spread $\boldsymbol{\Delta W/W}$

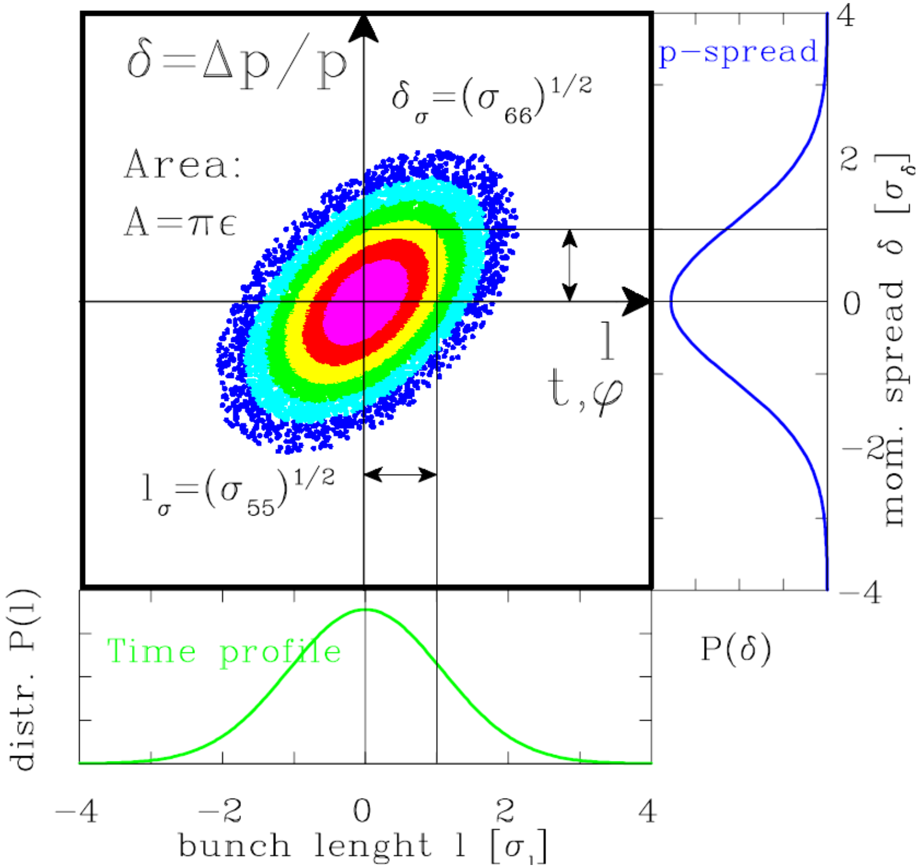
$$\Rightarrow \varepsilon_{long} = \frac{1}{\pi}\int_A dl \cdot d\delta$$

or with density function $\boldsymbol{\rho(l,\delta)}$

$$\Rightarrow \varepsilon_{long} = \frac{1}{\pi}\int \rho(l,\delta)\, dl \cdot d\delta$$

The normalized value is preserved:

$$\varepsilon_{long}^{norm} = \beta\gamma \cdot \varepsilon_{long}$$



Discussed devices:

- Pick-ups for bunch length and emittance.
- Other techniques: Special detectors (low $\boldsymbol{E_{kin}}$ protons), streak cameras (e^-)

The Bunch Position measured by a Pick-Up

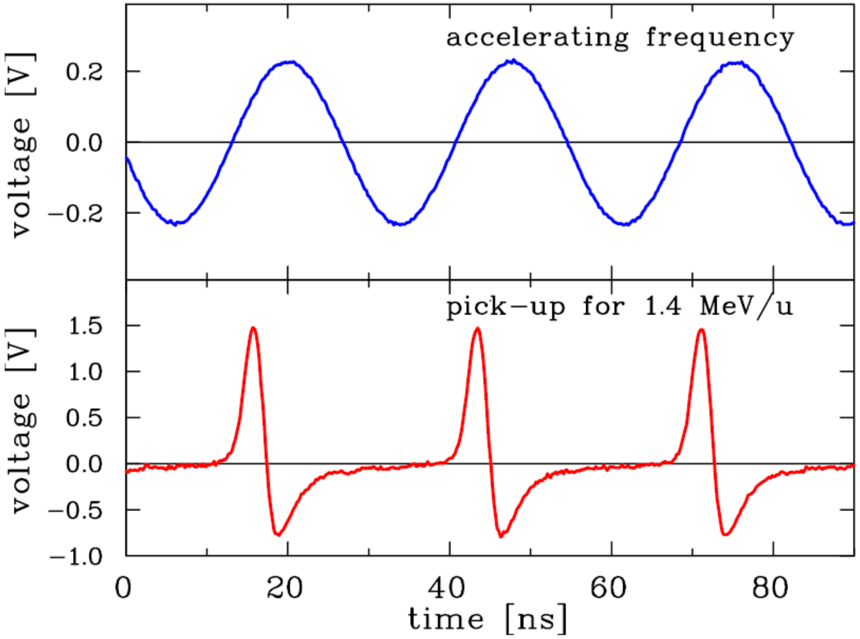
The ***bunch position*** is given relative to the accelerating rf.

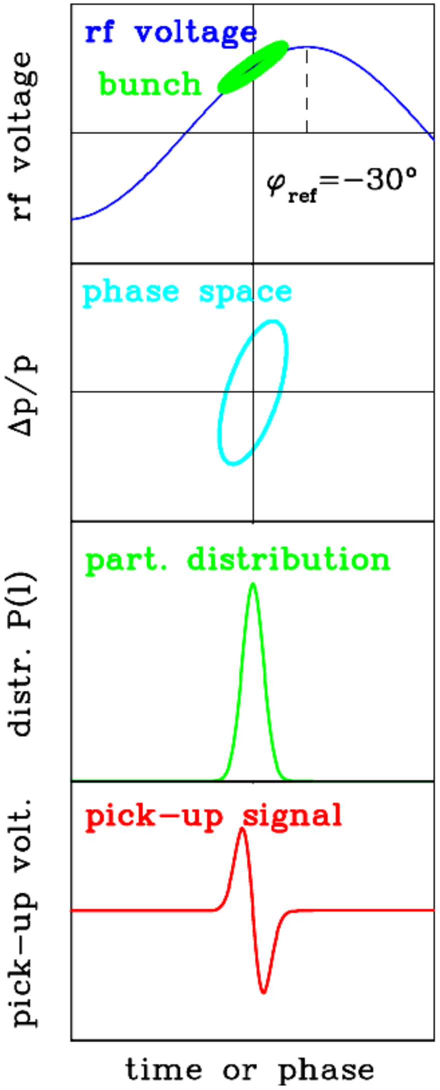
e.g. φ_{ref}=-30° inside a rf cavity

must be well aligned for optimal acceleration

Transverse correspondence: Beam position

Example: Pick-up signal and 36 MHz rf at GSI-LINAC:





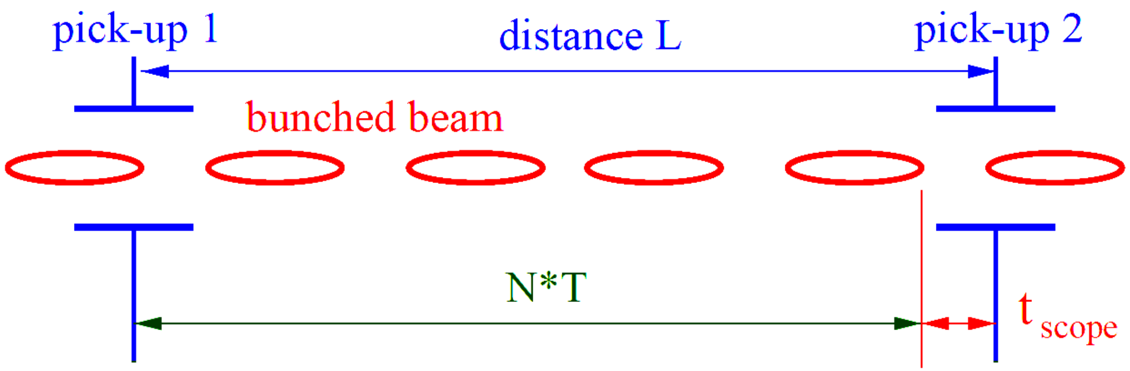
Determination of non-relativistic mean Energy using Pick-Ups

The energy delivered by a LINAC is sensitive to the mechanics, rf-phase and amplitude.

For non-relativistic energies at proton LINACs time-of-flight (TOF) with two pick-ups is used:

$$\beta c = \frac{L}{NT + t_{\text{scope}}}$$

→ the velocity ***β*** is measured.



Example: Time-of-flight signal from two pick-ups at 1.4 MeV/u:

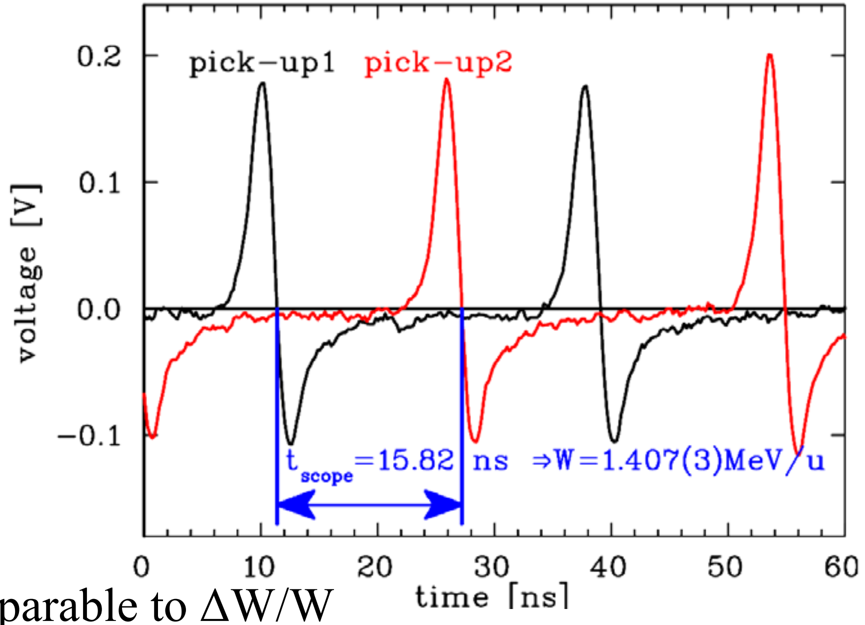
The reading is t_{scope}= 15.82(5)ns with

f_{rf} = 36.136MHz ⇔ ***T*** = 27.673ns

L = 1.629 m and ***N*** = 3

⇒ ***β*** = 0.05497(7)

⇒ ***W***=1.407(3) MeV/u



The accuracy is typically 0.1 % i.e. comparable to ΔW/W

Precision of TOF Measurement for non-relativistic Energy

The precision of TOF is given by the accuracy in time and distance reading:

$$\frac{\Delta\beta}{\beta} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta t}{NT + t_{\text{scope}}}\right)^2}$$

Accuracy of scope reading $\Delta t \approx 100$ ps, uncertainty in distance $\Delta L \approx 1$ mm.

Example: GSI-LINAC: $L = 3.25$ m and $f_{rf} = 36$ MHz:

location (GSI-slang)		RFQ	IH1	IH2	AL4
energy W	[MeV/u]	0.12	0.75	1.4	11.4
velocity β	%	1.6	4.0	5.5	15.5
total TOF	[ns]	677	271	197	70
bunch spacing $\beta c/f_{rf}$	[cm]	13	33	45	129
Number of bunches N		25	9	7	2
resolution $\Delta W/W$	%	0.07	0.10	0.12	0.22

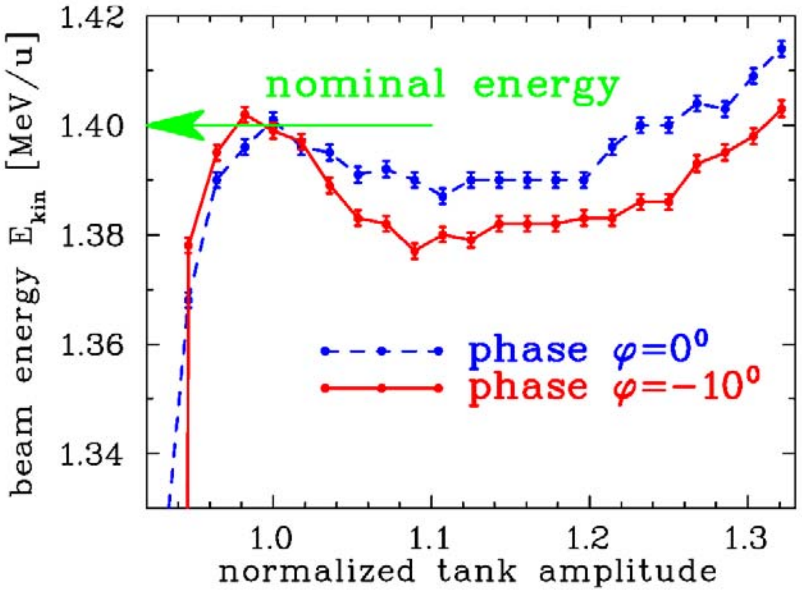
- The accuracy is typically 0.1 % (same order of magnitude as $\Delta W/W$)
- The length has to be matched to the velocity
- Due to the distance of ≈ 3 m, different solutions for the # of bunches N are possible

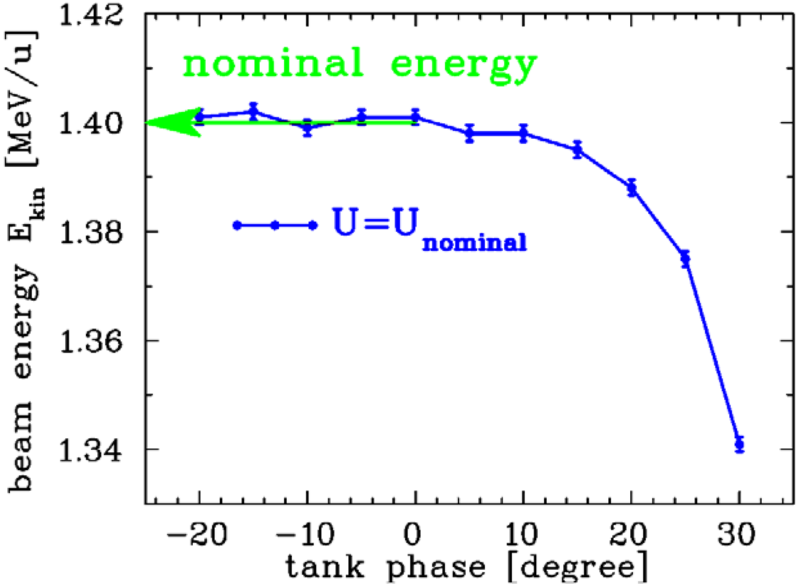
→ A third pick-up has to be installed closed by, to get an unique solution.

Cavity Alignment using a TOF Measurement

The mean energy is important for the matching between LINAC module.
It depends on phase and amplitude of the rf wave inside the cavities.

Example: Energy at GSI LINAC (nominal energy 1.400 MeV/u):
(distance between pick-ups: $\boldsymbol{L} = 1.97$ m $\Rightarrow \boldsymbol{N} = 4$ bunches)





- ➢***Proton LINACs:*** Amplitude and phase should be carefully aligned by precise TOF
- ➢***Electron LINACs***: Due to relativistic velocity, TOF is not applicable.

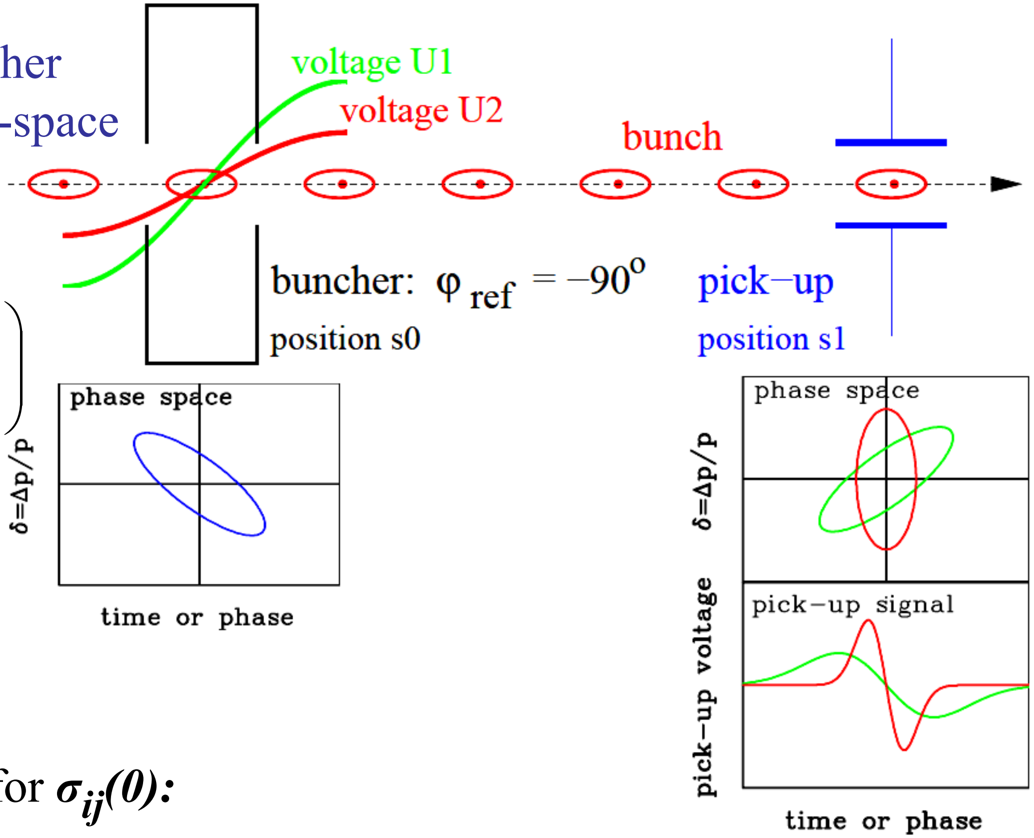
Longitudinal Emittance by linear Transformation using a Buncher

Longitudinal focusing:

Variation of the bunch shape by a rf-buncher

→ components 5 and 6 from 6-dim phase-space

Transversal corres.: Quadrupole variation



- Transfer matrix of buncher & drift:

$$R_{buncher} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \; R_{drift} = \begin{pmatrix} 1 & L/\gamma^2 \\ 0 & 1 \end{pmatrix}$$

with focal length: $1/f = \frac{2\pi f_{rf}}{Apv^2} \cdot U$

- Variation of buncher amplitude ***U***

  ⇒ different bunch width at s_1:

  beam matrix $\Delta t^2_{max} = \sigma_{55}(1, f)$

- System of redundant linear equations for $\sigma_{ij}(0)$:

$$\begin{aligned} \sigma_{55}(1, f_1) &= R_{55}^2(f_1) \cdot \sigma_{55}(0) + 2R_{55}(f_1)R_{56}(f_1) \cdot \sigma_{56}(0) + R_{56}^2(f_1) \cdot \sigma_{66}(0) && \text{focusing } f_1 \\ &\vdots \\ \sigma_{55}(n, f_n) &= R_{55}^2(f_n) \cdot \sigma_{55}(0) + 2R_{55}(f_n)R_{56}(f_n) \cdot \sigma_{56}(0) + R_{56}^2(f_n) \cdot \sigma_{66}(0) && \text{focusing } f_n \end{aligned}$$

6-dim Phase Space for Accelerators

The particle trajectory is described with the 6-dim vector $\vec{x}^{\,t} = \left(x, x', y, y', l, \delta\right)$

For linear beam behavior the 6x6 transport matrix R is used:

The transformation from location s_0 to s_1 is:

$$\vec{x}(s_1) = R \cdot \vec{x}(s_0)$$

$$= \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & \dots \;{=}0 & \dots & \dots \;{=}0 & \dots \\ R_{31} & \dots & R_{33} & R_{34} & \dots & \dots \\ R_{41} \;{=}0 & \dots & R_{43} & R_{44} & \dots \;{=}0 & \dots \\ R_{51} & \dots & \dots & \dots & R_{55} & R_{56} \\ R_{61} \;{=}0 & \dots & \dots \;{=}0 & \dots & R_{65} & R_{66} \end{pmatrix} \cdot \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

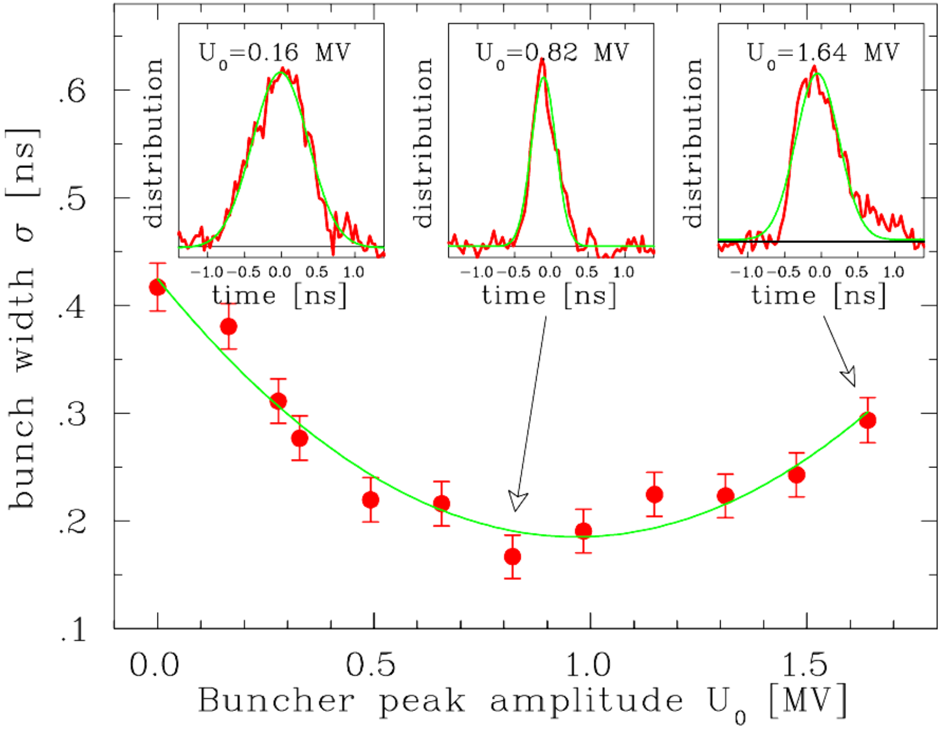
R separates in 3 matrices only if the horizontal, vertical and longitudinal planes do **not** couple,

e.g. no dispersion $D = -R_{16} = 0$

Result of a longitudinal Emittance Measurement

Example GSI LINAC:

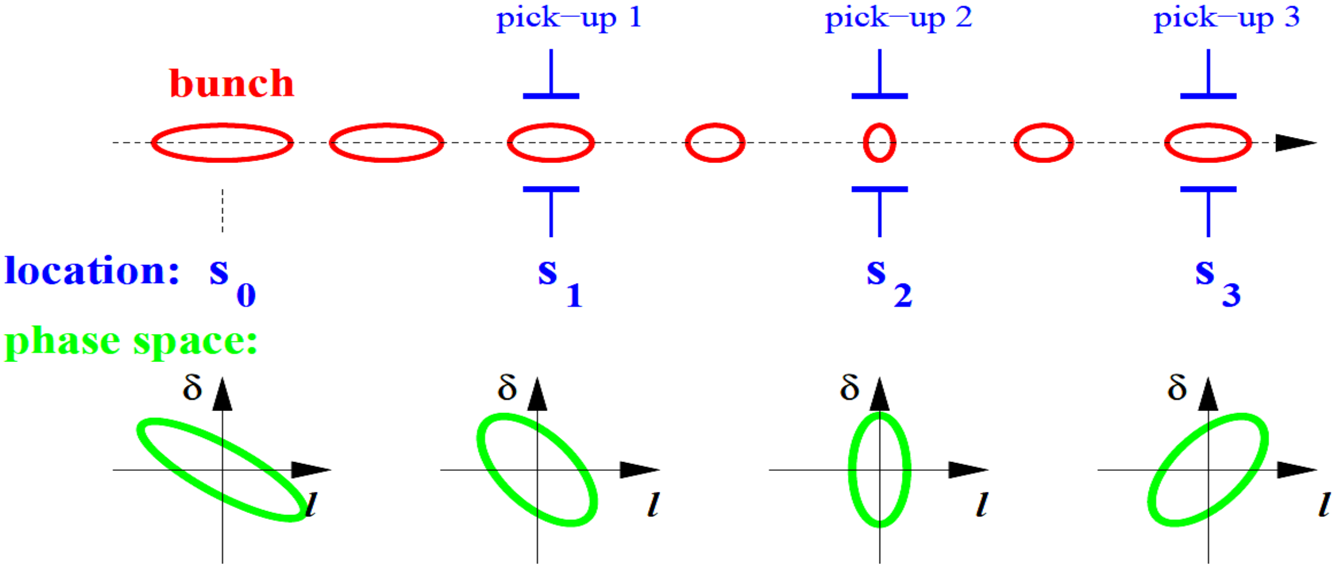
The voltage at the single gap -resonator is varied for 11.4 MeV/u Ni^{14+} beam, 31 m drift:



- ➢ The structure of short bunches can be determined with special monitor
- ➢ This example: The resolution is better than 50 ps or 2^o for 108 MHz
- ➢ Typical bunch length at proton LINACs: 30 to 200 ps

Longitudinal Emittance within a Transfer-Line

As for the 'three grid' method, the emittance can be determined in a transfer line.



The system of redundant linear equations with the transfer matrix **R(i)** to location ***s**$_i$*:

$$\begin{aligned}
\sigma_{55}(1) &= R_{55}^2(1)\cdot\sigma_{55}(0) + 2R_{55}(1)R_{56}(1)\cdot\sigma_{56}(0) + R_{56}^2(1)\cdot\sigma_{66}(0) \qquad \mathbf{R}(1): s_0 \rightarrow s_1 \\
&\vdots \\
\sigma_{55}(n) &= R_{55}^2(n)\cdot\sigma_{55}(0) + 2R_{55}(n)R_{56}(n)\cdot\sigma_{56}(0) + R_{56}^2(n)\cdot\sigma_{66}(0) \qquad \mathbf{R}(n): s_0 \rightarrow s_n
\end{aligned}$$

Assumptions:
- Bunches much longer than pick-up or relativistic E -field: $E_\perp \gg E_{||}$
- Gaussian distribution without space-charge effects.

Longitudinal Emittance using tomographic Reconstruction

Tomography is medical image method

Tomography:
2-dim reconstruction of
sufficient 1-dim projections

Application at accelerators:

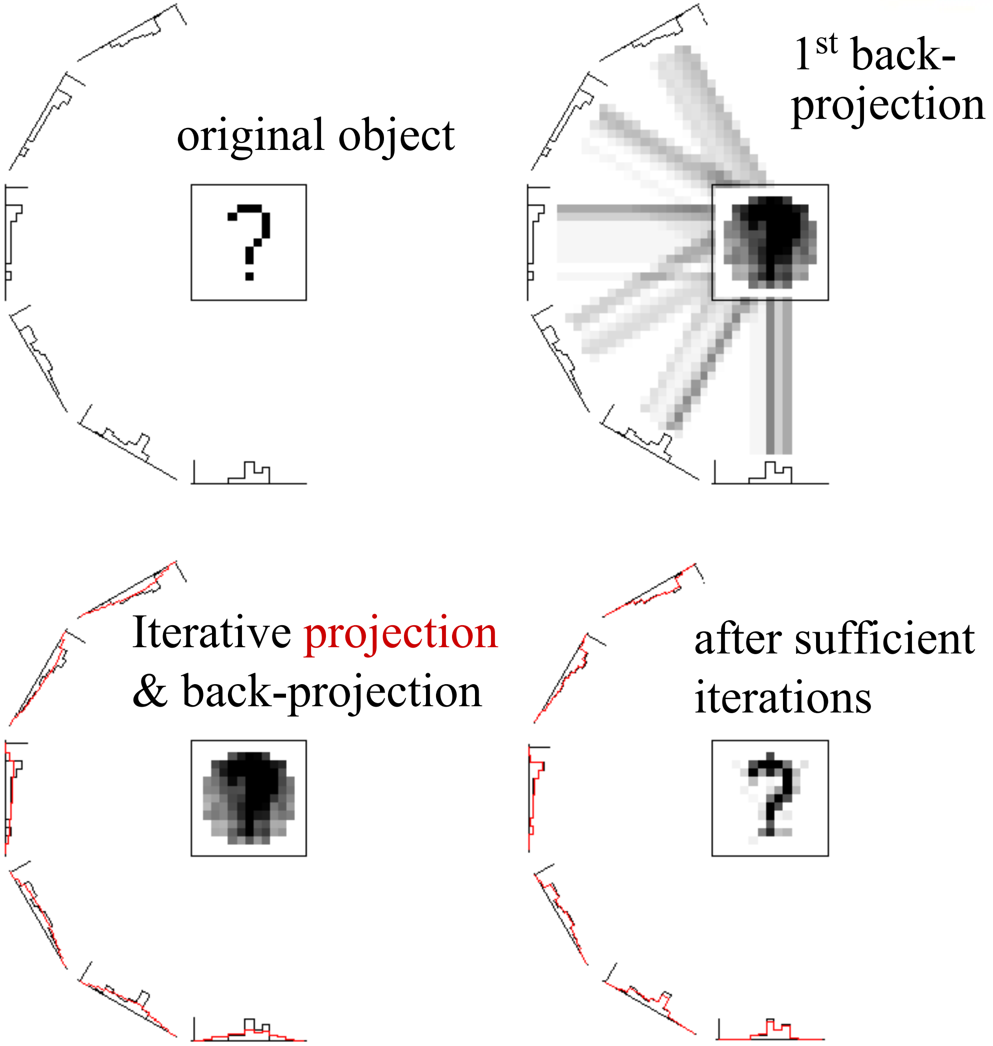
Longitudinal emittance
evolution in synchrotrons.

Bunch observation:

Each revolution,
the bunch shape changes a bit
due to synchrotron oscillations.
Fulfilled condition: $f_{synch} << f_{ref}$.

Algebraic back projection:

Iterative process by redistributing
the 2-dim image and considering the
differences to the previous iteration step.



original object

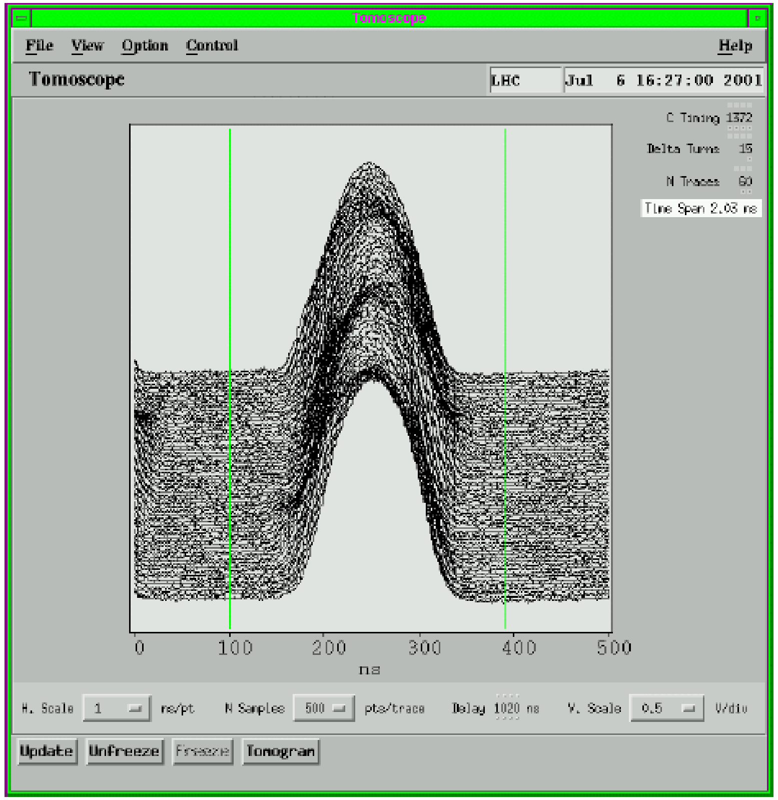
1^{st} back-projection

Iterative projection & back-projection

after sufficient iterations

Results of tomographic Reconstruction at a Synchrotron I

Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



LHC Jul 6 16:27:00 2001

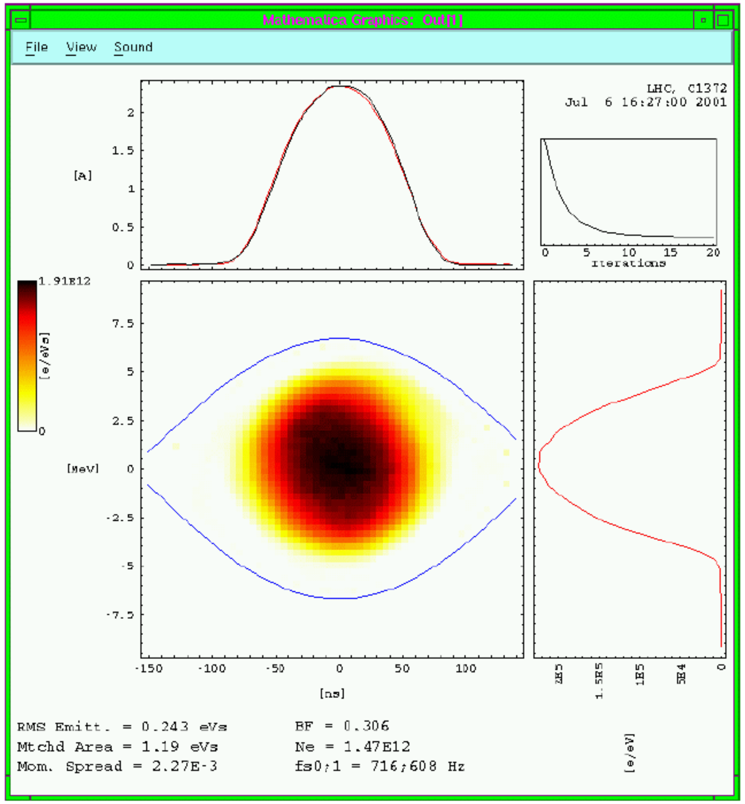
C Timing 1372

Delta Turns 15

N Traces 60

Time Span 2.03 ns

H. Scale 1 ns/pt N Samples 500 pts/trace Delay 1020 ns V. Scale 0.5 V/div



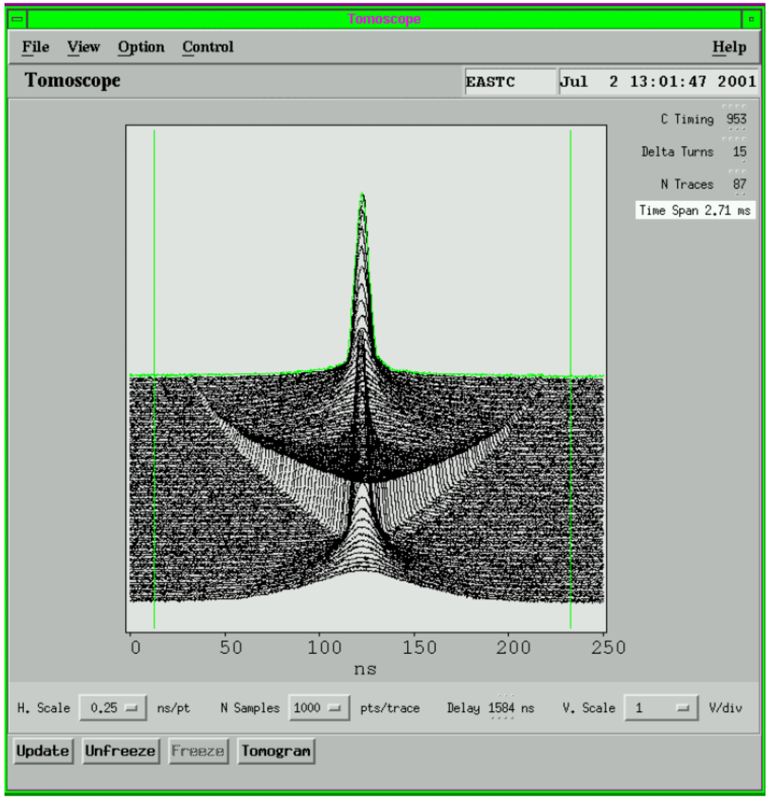
LHC, C1372
Jul 6 16:27:00 2001

RMS Emitt. = 0.243 eVs
Mtchd Area = 1.19 eVs
Mom. Spread = 2.27E-3
BF = 0.306
Ne = 1.47E12
fs0;1 = 716;608 Hz

Typical bucket filling. Important knowledge for bunch 'gymnastics'.

Results of tomographic Reconstruction at a Synchrotron II

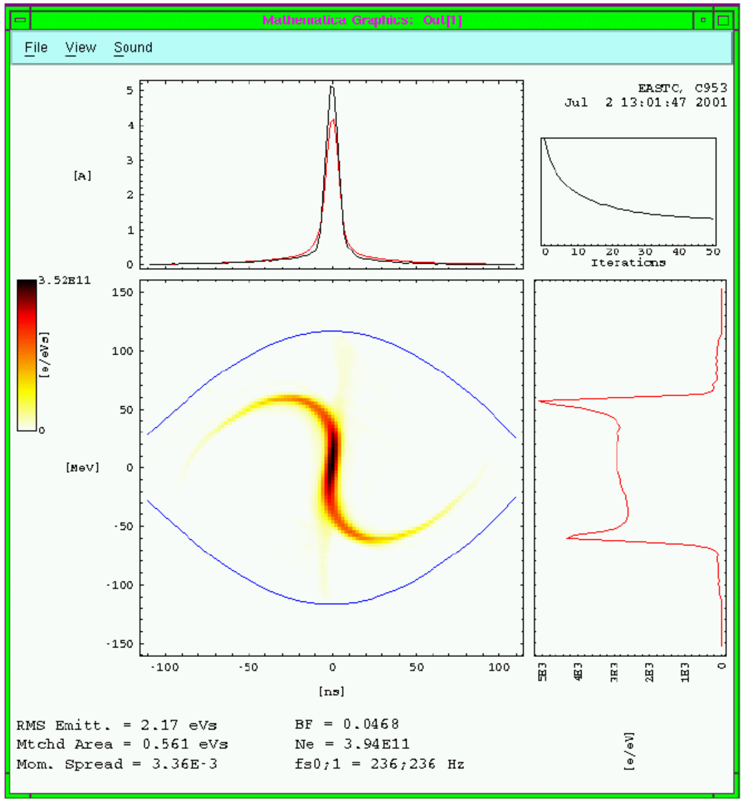
Bunches from 500 turns at the CERN PS and the phase space for the first time slice, measured with a wall current monitor:



EASTC Jul 2 13:01:47 2001

C Timing 953
Delta Turns 15
N Traces 87
Time Span 2.71 ms

H. Scale 0.25 ns/pt
N Samples 1000 pts/trace
Delay 1584 ns
V. Scale 1 V/div



EASTC, C953
Jul 2 13:01:47 2001

RMS Emitt. = 2.17 eVs
Mtchd Area = 0.561 eVs
Mom. Spread = 3.36E-3
BF = 0.0468
Ne = 3.94E11
fs0;1 = 236;236 Hz

Mismatched bunch shown oscillations and filamentation due to ‘bunch-rotation’.

Resistive Wall Current Monitor

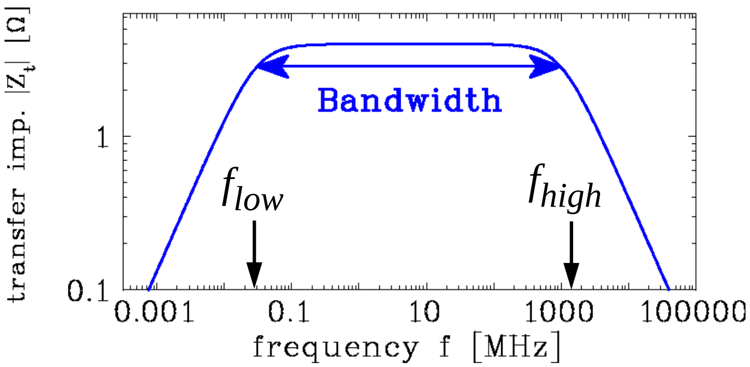
Broadband observation of bunches can be performed with a resistive Wall Current Monitor

Principle:
- Ceramic gap bridged with n =10…100 resistors of R = 10…100 Ω
- Measurement of voltage drop for $R_{tot} = R/n$ = 1…10 Ω
- Ferrite rings with high L → forces low frequency components through resistors

Bandwidth: typically $f_{low} = R_{tot}/(2\pi L) \approx$ **10 kHz**

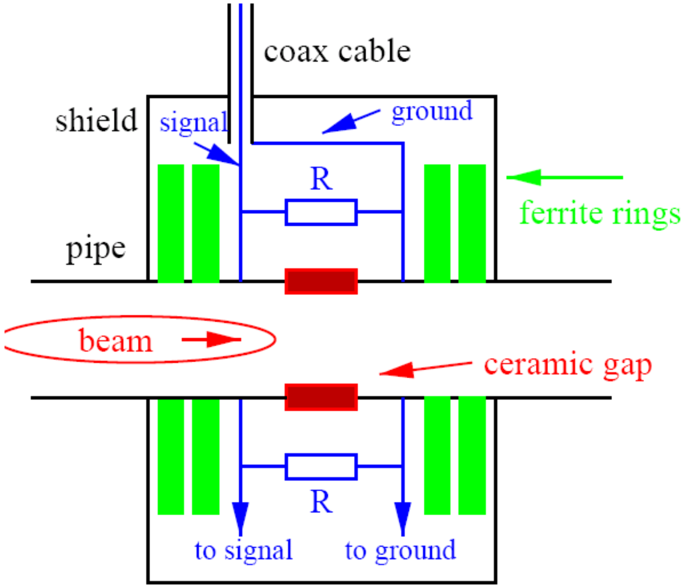
$f_{high} = 1/(2\pi R_{tot} C) \approx$ **1 GHz**

Application: Broadband bunch observation.

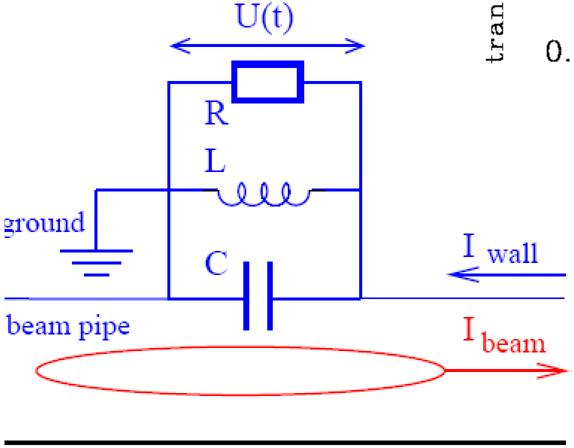


$$\frac{1}{Z_t} = \frac{1}{R_{tot}} + \frac{1}{i\omega L} + i\omega C$$

Within bandwidth: $Z_t \cong R_{tot}$

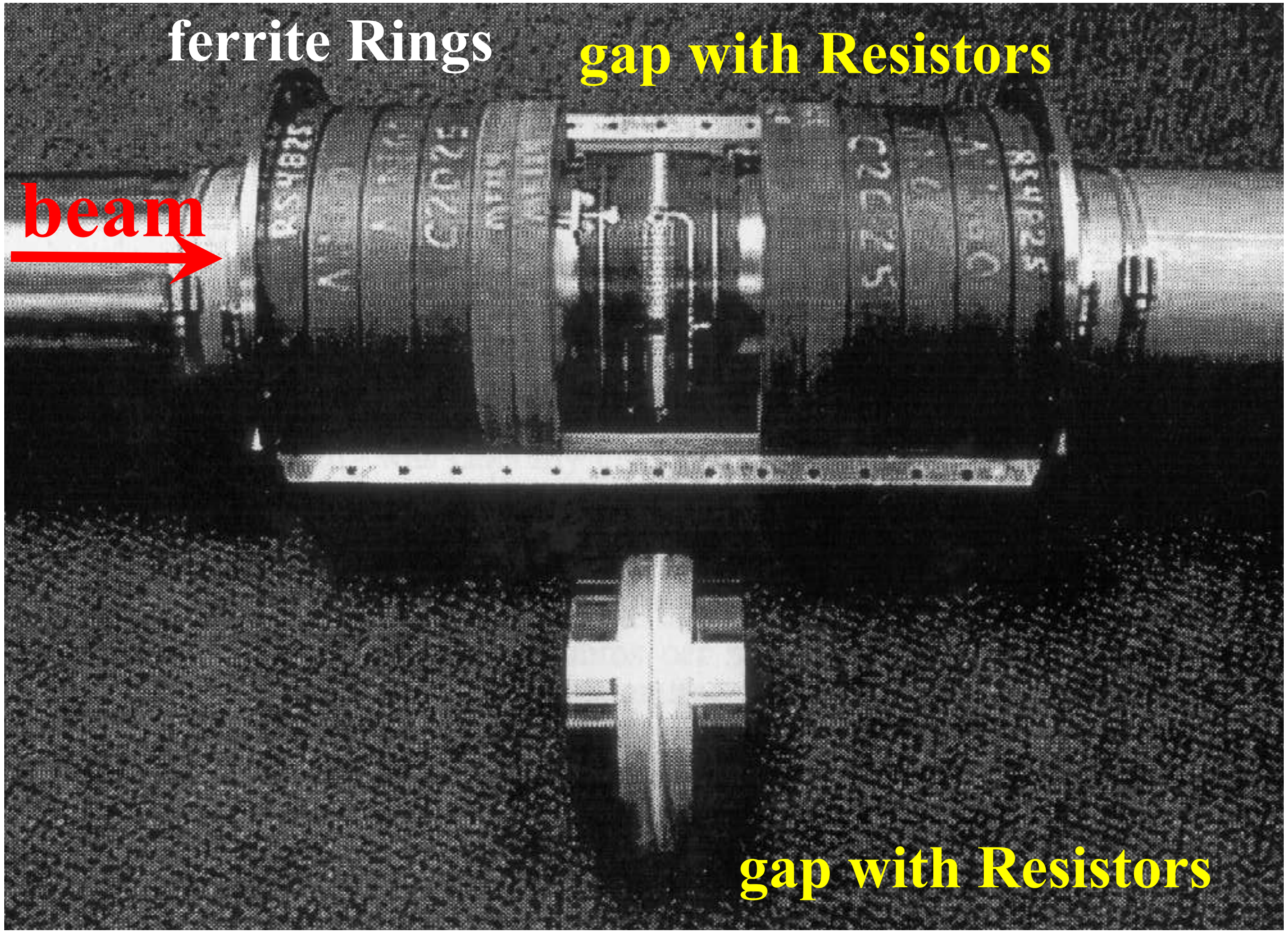


WCM equivalent circuit



Resistive Wall Current Monitor

Example: Realization at Fermi-Laboratory



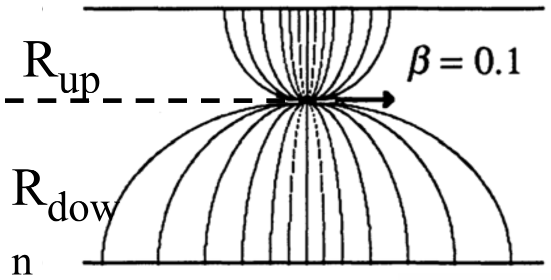
Bunch Structure at low E_{kin}: Not possible with Pick-Ups

Pick-ups are used for:

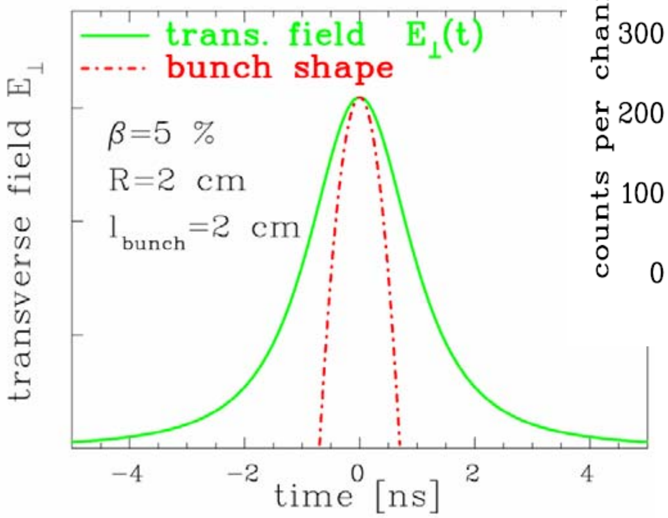
- precise for bunch-center relative to rf
- course image of bunch shape

But:

For $\beta \ll 1 \Rightarrow$ long. E-field significantly modified:

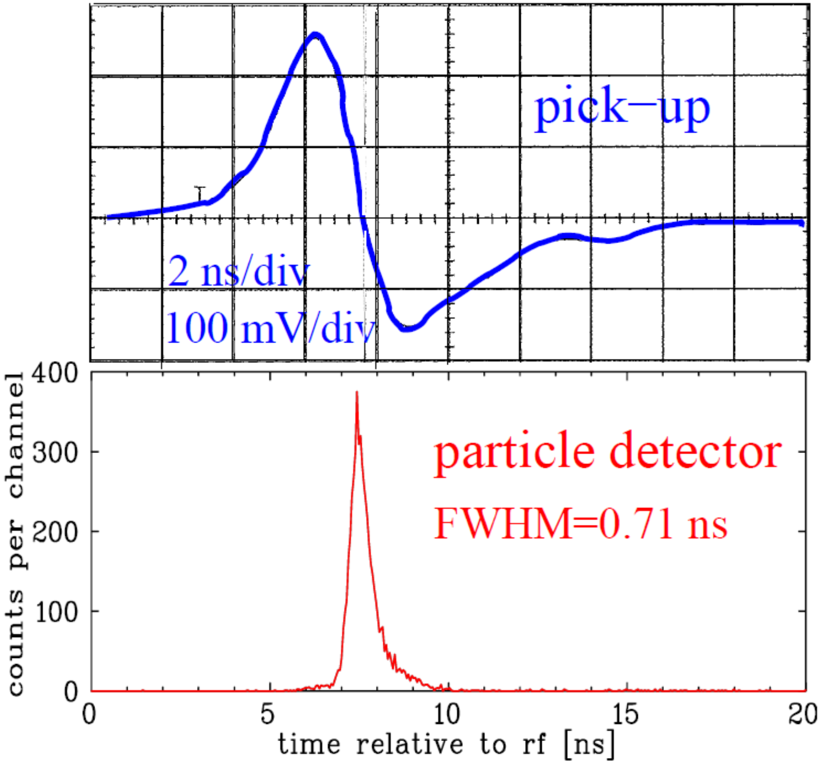


ampl.



Example: Comparison pick-up – particle counter:

Ar^{1+} with 1.4 MeV/u (β = 5.5%)



$\Rightarrow$ the pick-up signal is insensitive to bunch 'fine-structure'

Low Velocity Effect: General Consideration

Lorentz transformation of single point-like charge:

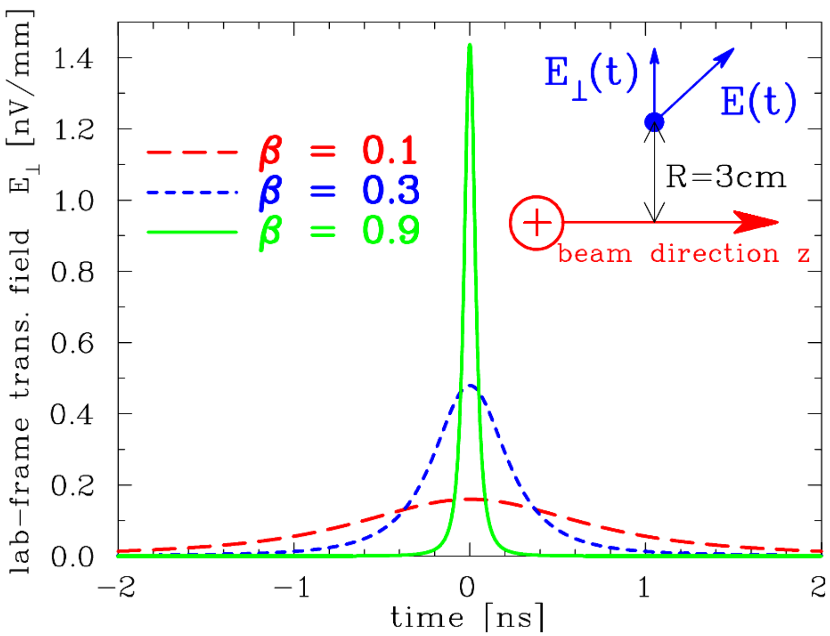
Lorentz boost ***and*** transformation of time: $E_\perp(t) = \gamma \cdot E'_\perp(t')$ and $t \rightarrow t'$

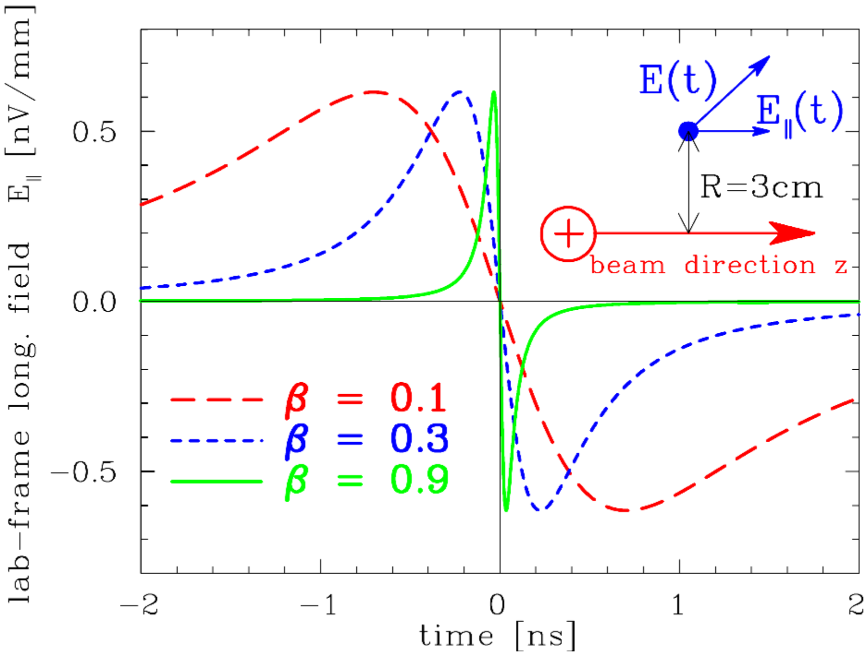
Trans. $\boldsymbol{E}_\perp$ lab.-frame of a point charge:

$$E_\perp(t) = \frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma R}{\left[R^2 + (\gamma\beta c t)^2\right]^{3/2}}$$

Long. $\boldsymbol{E}_{||}$ lab.-frame of a point charge:

$$E_{||}(t) = -\frac{e}{4\pi\varepsilon_0} \cdot \frac{\gamma\beta c t}{\left[R^2 + (\gamma\beta c t)^2\right]^{3/2}}$$

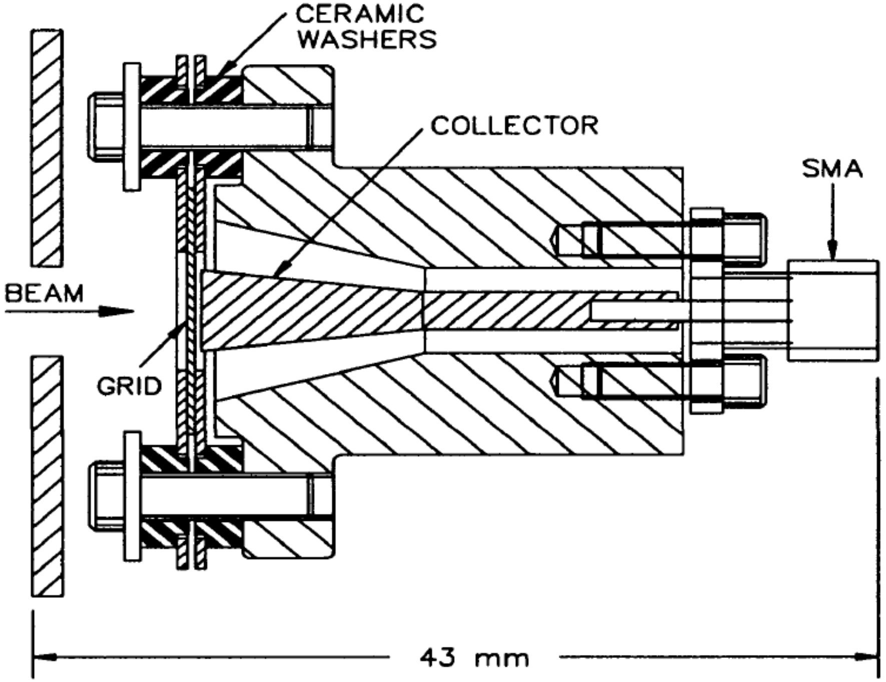




Broadband coaxial Faraday Cups for Bunch Structure

The bunch structure can be observed with cups, having a bandwidth up to several GHz.

Bandwidth and rise time: BW [GHz] = **0.3/t_{rise}**[ns]



Impedance of a

coaxial transmission line:

$$Z_0 = \frac{Z_c}{2\pi} \cdot \ln \frac{r_{outer}}{r_{inner}}$$

$$\text{with } Z_c = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

⇒ impedance matching to prevent for reflections

Voltage reflection: $\rho_V = \frac{Z - Z_0}{Z + Z_0}$

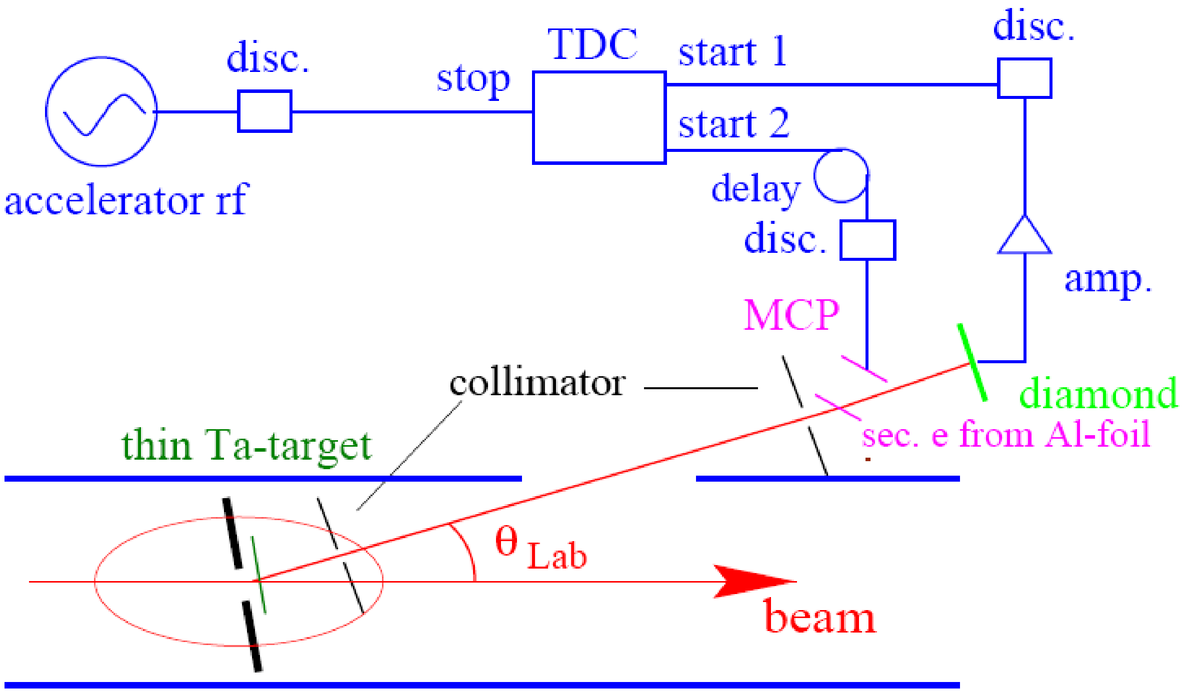
Voltage Standing Wave Ratio: $\text{VSWR} = \frac{Z}{Z_0} = \frac{1+\rho_V}{1-\rho_V}$

$\boldsymbol{Z = Z_0}$: no reflection. $\boldsymbol{Z = 0} \Rightarrow \boldsymbol{\rho_V} = -1$: short circuit. $\boldsymbol{Z} = \infty \Rightarrow \boldsymbol{\rho_V} = 1$: open circuit.

Realization of a Broadband coaxial Faraday Cup

Time-of-Flight using Particle Detectors

The time of arrival of the particle is determined relative to the accelerating rf:



Realization at GSI-LINAC: Less than one particle per bunch due to ***single*** particle counting:

→ **Foil (130 nm):** attenuation ≈10^{-9} by Rutherford scat. ⇒finite solid angle $\Delta\Omega_{lab} = 2.5 \cdot 10^{-4}$

→ **Stop-detectors:** Fast detector with 1 ns pulse width (diamond)

→ **TDC:** Time relative to rf, resolution less than 25 ps (corresponding to 0.3^o in phase)

→ **Start-detector**: 2nd thin Al foil (50 nm) for secondary e^- acc. toward an MCP +50 Ω anode

⇒ **Result:** determination of phase and energy of ***individual*** particles.

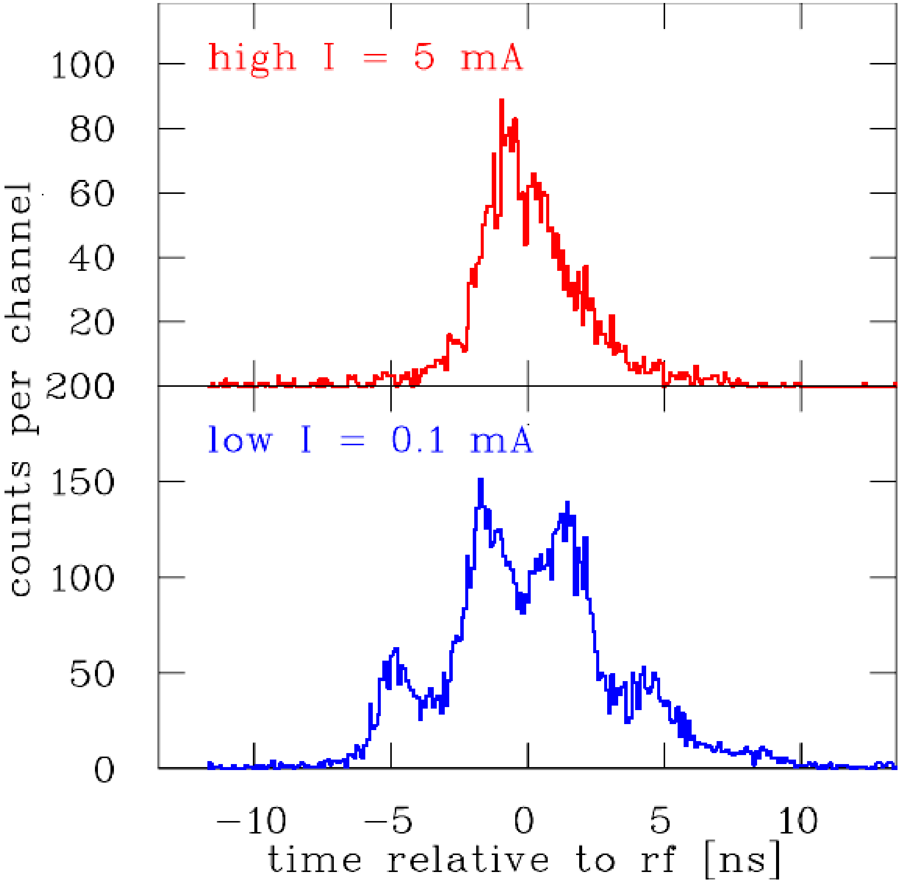
Result of Bunch Structure Determination at low E_{kin}

The bunch structure is dependent on the amplitude or phase setting

→ wrong bunching (RFQ), emittance blow-up, filamentation...

→ non-Gaussian distributions are possible

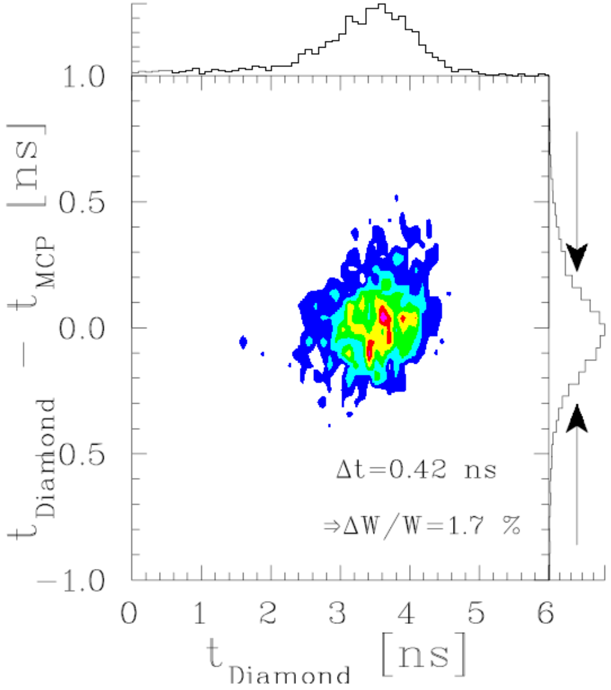
Example: The bunch shape at 120 keV/u from GSI-LINAC with different currents:

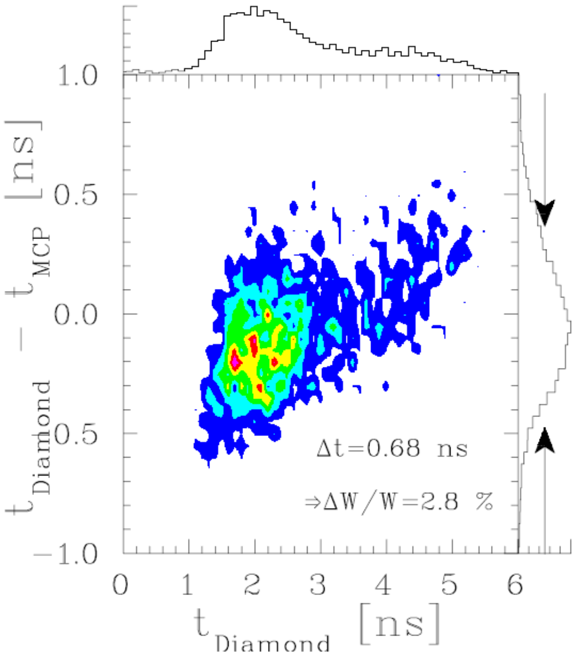


Results of longitudinal Emittance Determination at low E_{kin}

No 'standard' method for longitudinal emittance measurement is available!
Using two detectors in coincidence and a drift space in between,
the phase and the energy of a individual particle can be determined.
⇒ for many particles the longitudinal phase space can be spanned.

Example: GSI-LINAC at 1.4 MeV/u with ***low*** and ***high*** current Ar beam
The effect of the emittance blow-up due to the large space-charge is seen.





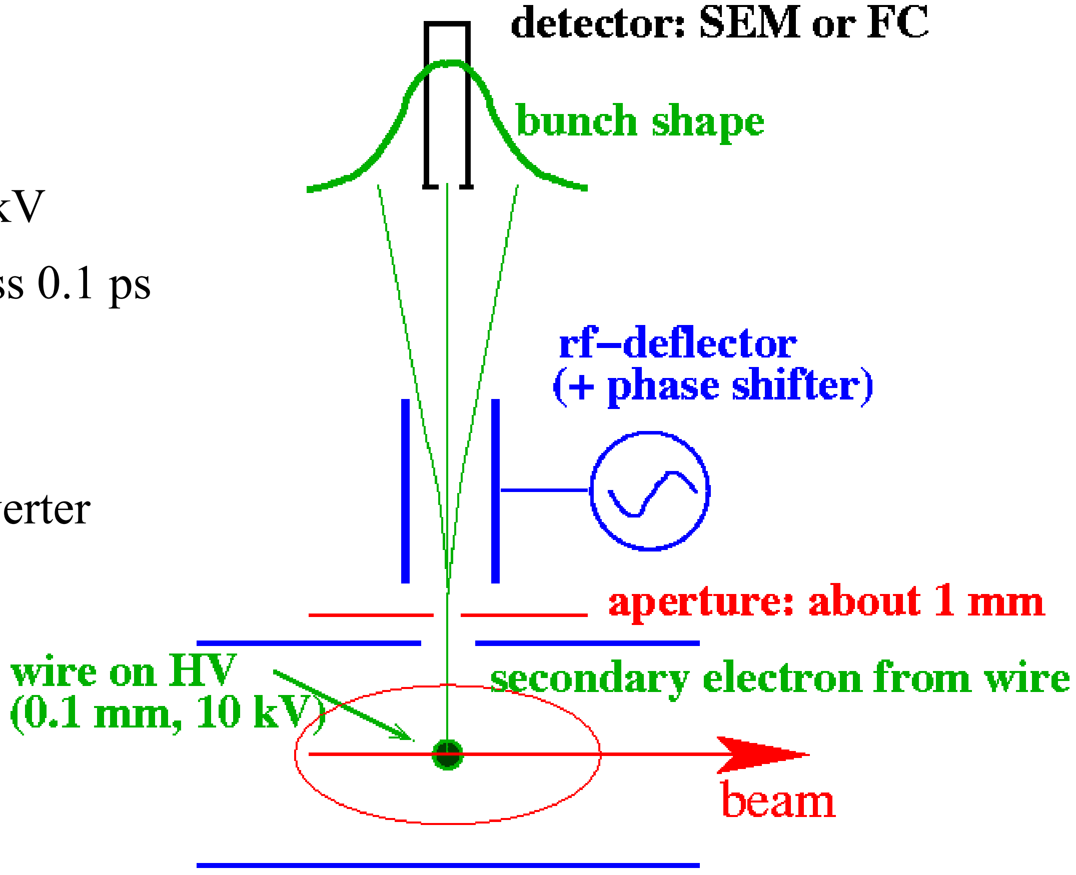
Bunch Structure using secondary Electrons for low E_{kin} Protons

Secondary e^- liberated from a wire carrying the time information.

→ Bunch Shape Monitor (BSM)

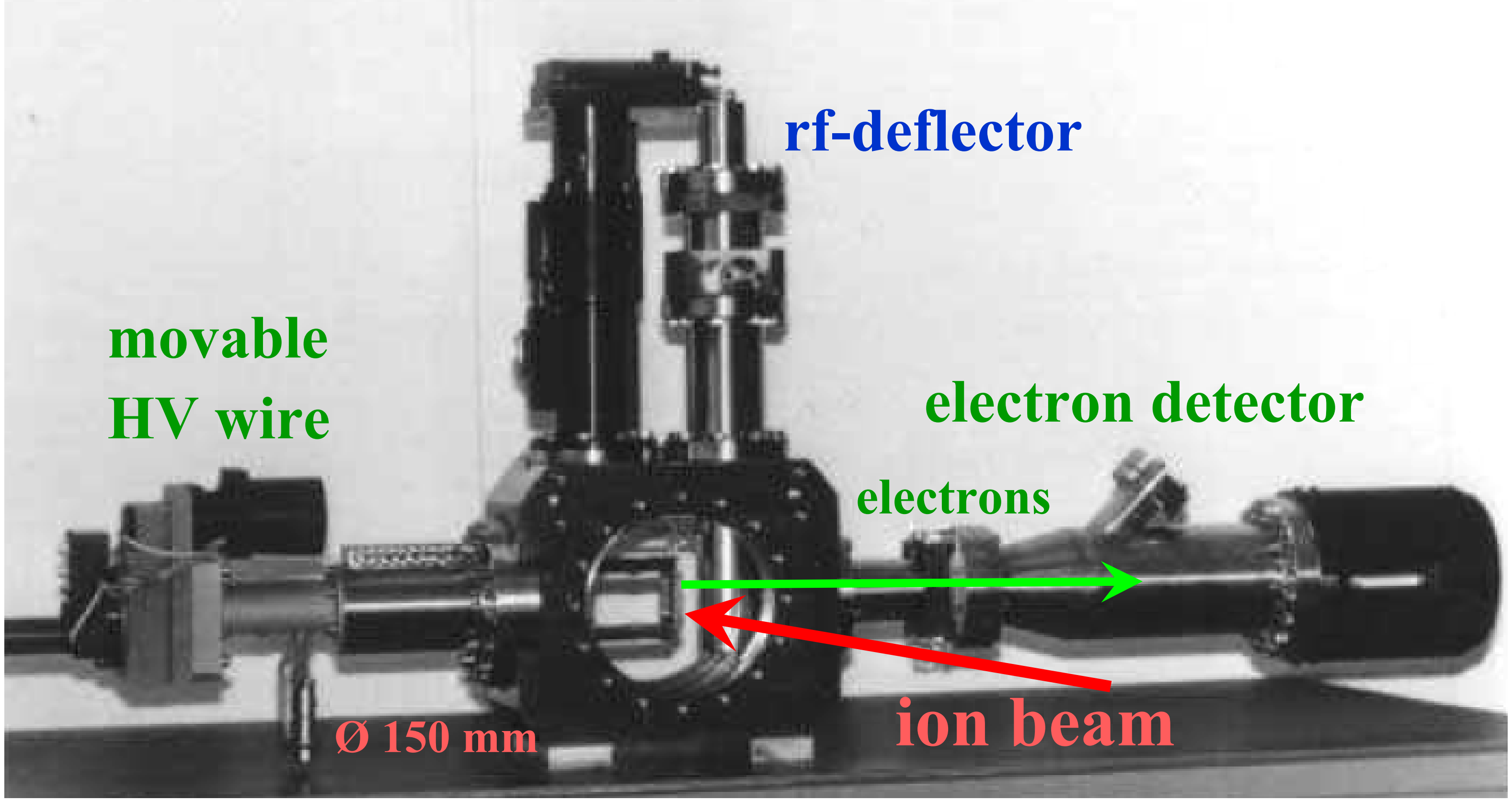
Working principle:

- insertion of a 0.1 mm wire at ≈ 10 kV
- emission of secondary e^- within less 0.1 ps
- secondary e^- are accelerated
- toward an rf-deflector
- rf-deflector as ’time-to-space’ converter
- detector with a thin slit
- slow shift of the phase
- resolution $\approx 1^o < 10$ ps
- Measurements are comparable to that obtained with particle detectors.



SEM: secondary electron multiplier

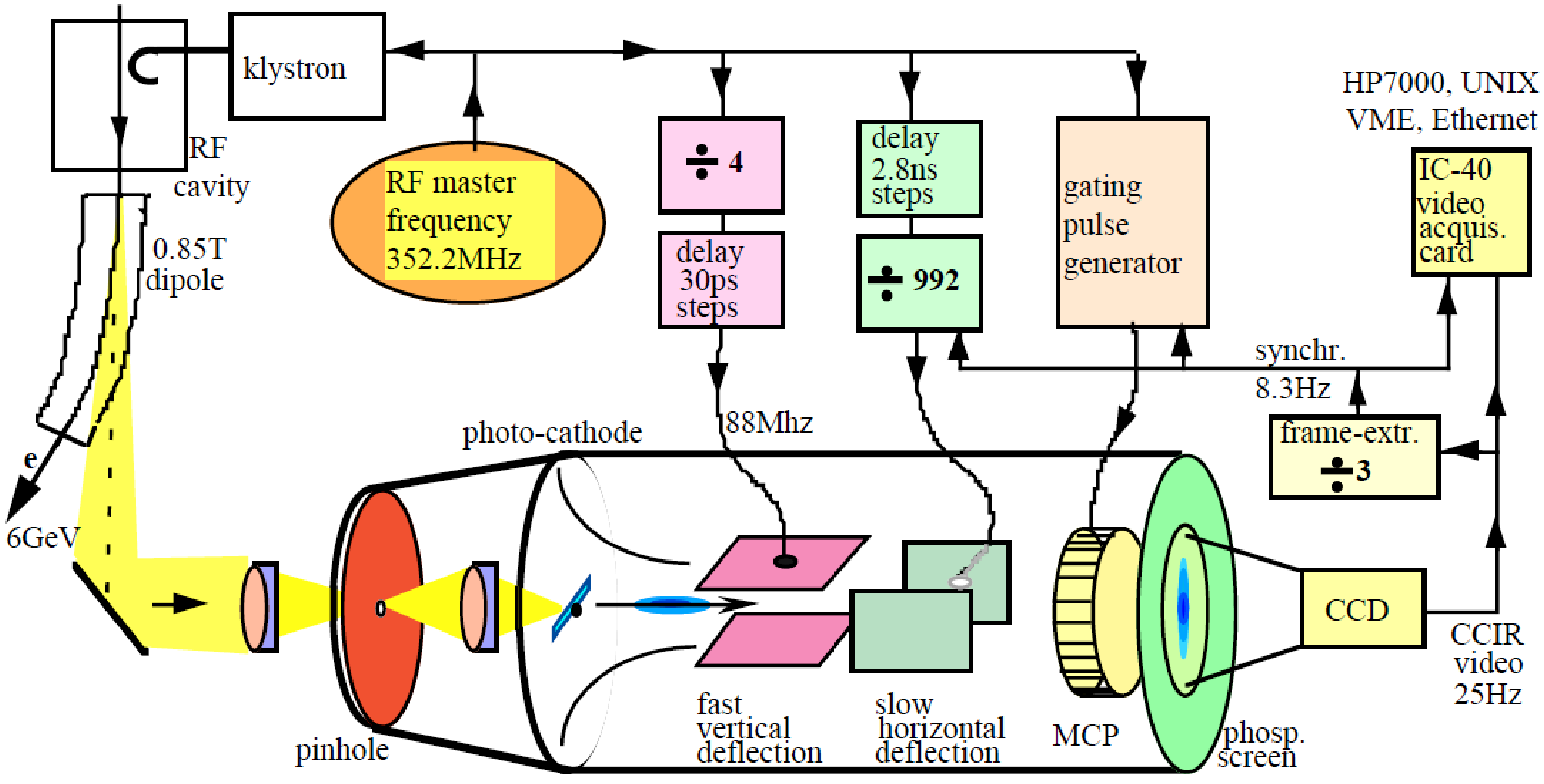
Realization of Bunch Shape Monitor at CERN LINAC2



Bunch Length Measurement for relativistic e^-

Electron bunches are too short ($\sigma_t < 300$ ps) to be covered by the bandwidth of pick-ups ($f < 1$ GHz ⇔ $t_{rise} > 300$ ps) for structure determination.

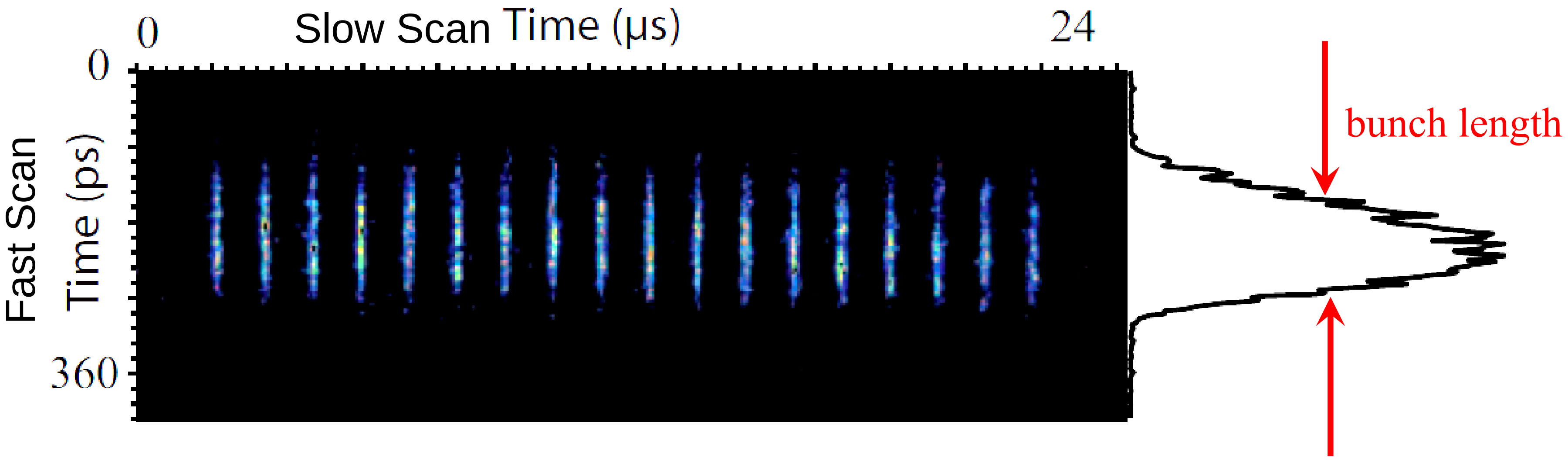
→ Time resolved observation of synchr. light with a streak camera: Resolution ≈ 1 ps.



Results of Bunch Length Measurement by a Streak Camera

The streak camera delivers a fast scan in vertical direction (here 360 ps full scale) and a slower scan in horizontal direction (24 μs).

Example: Bunch length at the synch. Light source SOLEIL for $\boldsymbol{U_{rf}}$=2MV
for slow direction 24 μs and scaling for fast scan 360 ps ⇒ $\boldsymbol{\sigma_t}$ = 35 ps.



The Artist View of a Streak Camera

FARADAY CUP 1998

Purpose. To recognize and encourage innovative achievements in the field of accelerator beam instrumentation.

Award. The Faraday Cup Award consists of a US$ 5000 prize and a certificate to be presented at the next Beam Instrumentation Workshop. Winners participating in the BIW will be given a $1000 travel allowance.

Eligibility. Nominations are open to contributors of all nations regardless of the geographical location at which the work was done.
The Award goes normally to one person, but may be shared by recipients having contributed to the same accomplishment. It will normally be awarded to scientists in the early stage of their career. Nominations of candidates shall remain active for 2 competitions.

Establishment and support. The Award was established in 1991 with the support of the Beam Instrumentation Workshop Organizing Committee.

Rules. The Faraday Cup shall be awarded for an outstanding contribution to the development of an innovative beam diagnostics instrument of proven workability. The Faraday Cup is only awarded for published contribution and delivered performance –as opposed to theoretical performance. Rules are available on request.

Award Committee. The Beam Instrumentation Workshop Organizing Committee.

Nominations. The nomination package shall include the name of the candidate, relevant publications, a statement outlining his/her personal contribution and that of others, two letters from co-workers familiar with the candidate and his contribution. Two master copies suitable for photocopying of this package must be submitted not later than the 15th of November 1997 to Steven Smith c/o BIW'98 Secretariat, SLAC, Stanford University, Stanford CA 94305-4085, U.S.A.

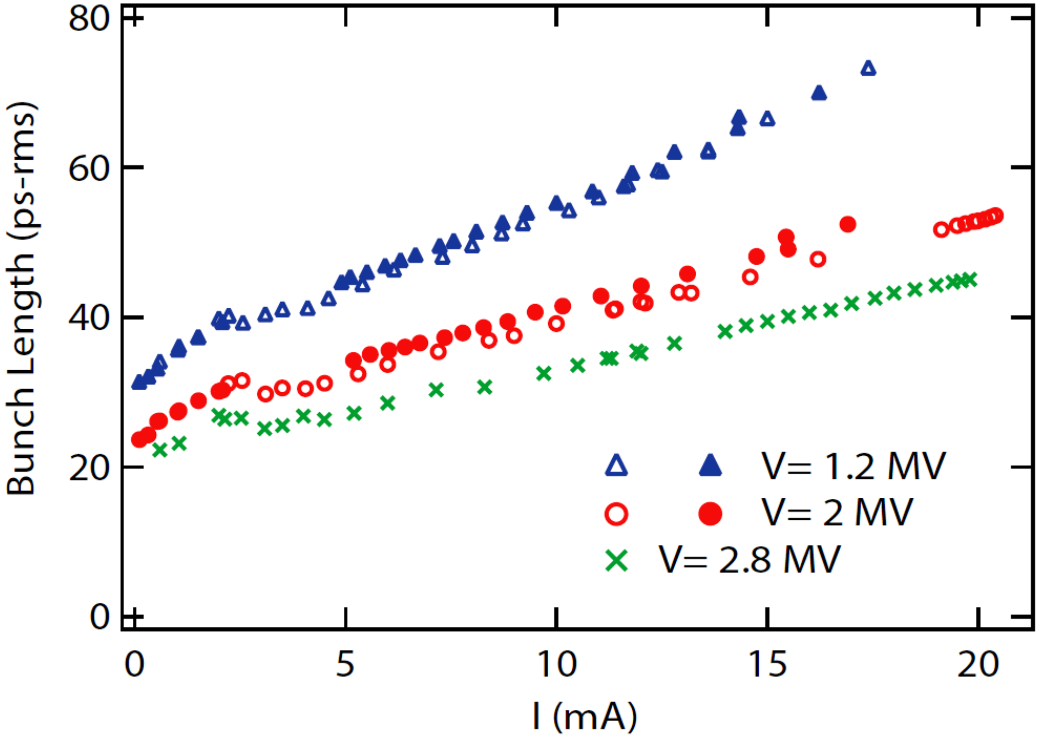
The Importance of Bunch Length by Streak Camera

Short bunches are desired by the synchrotron light users for time resolved spectroscopy.

The bunch focusing is changed by the rf-amplitude.

Example: Bunch length σ_t as a function of stored current

(space-charge de-focusing, impedance broadening) for different rf-amplitudes at SOLEIL:



Measurement of Beam Profile

For Free Electron Lasers → bunch length below 1 ps is used

→ below resolution of streak camera

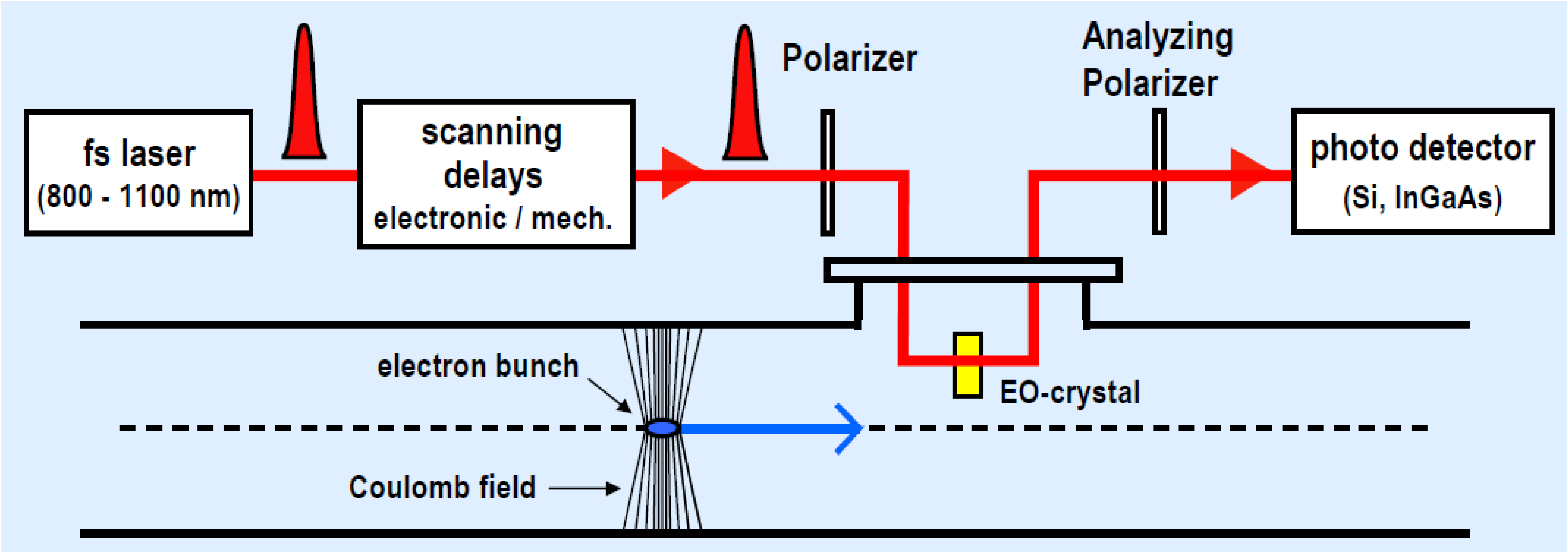
→ short laser pulses with ***t*≈10 fs and** electro-optical modulator

Electro optical modulator: birefringent, rotation angle depends on external electric field

Relativistic electron bunches: transverse field $\boldsymbol{E}_{\perp,\,lab} = \gamma \boldsymbol{E}_{\perp,\,rest}$ carries the time information

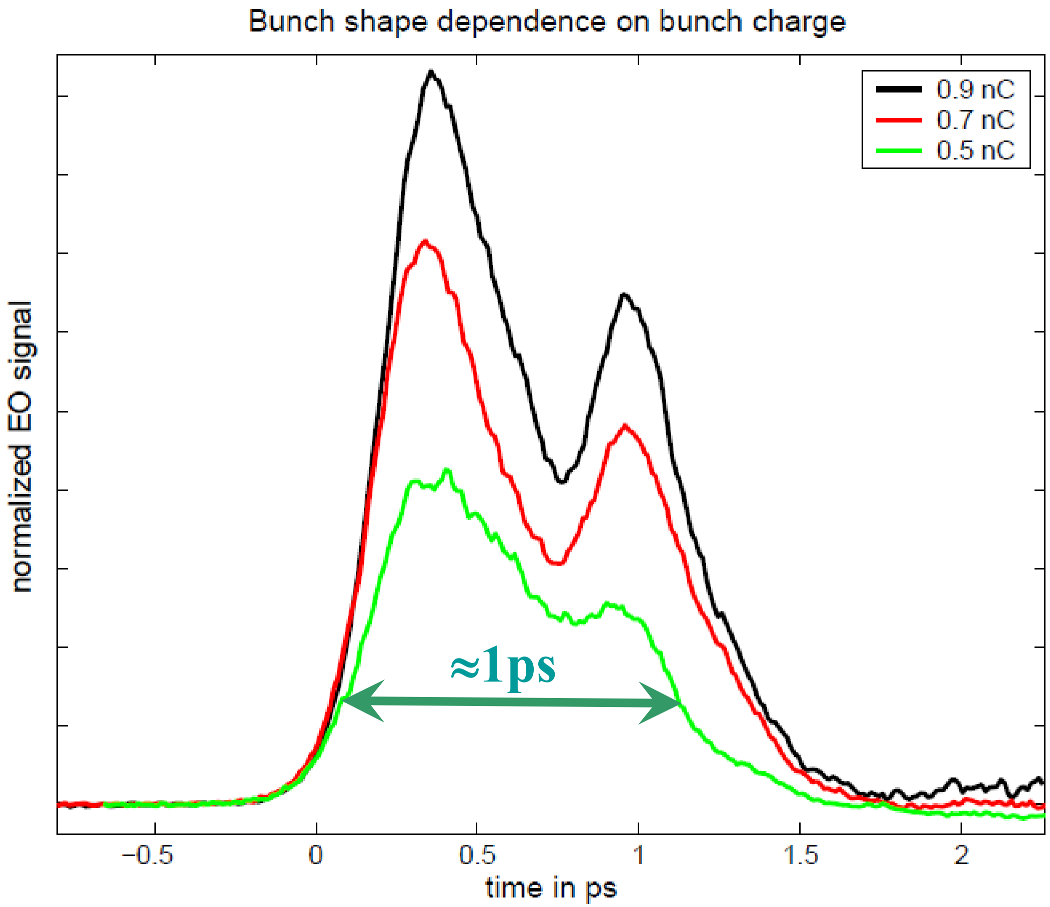
Scanning of delay between bunch and laser → time profile after several pulses

Additionally, single shot modifications successfully tested.

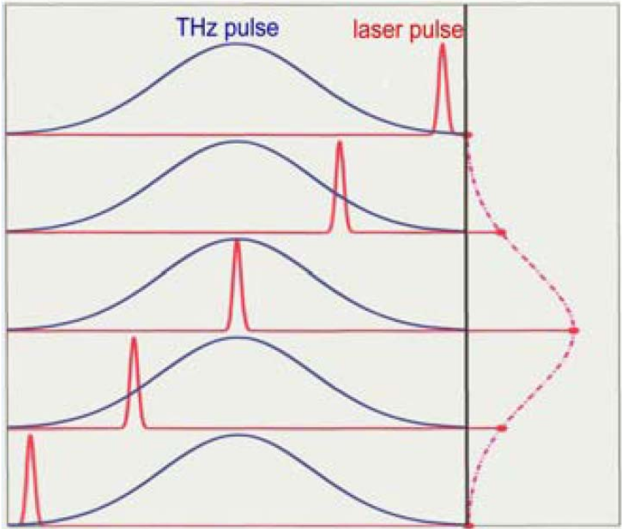


Measurement of Bunch Shape at FEL-Facility

Example: Bunch length at FEL test facility FLASH



Scanning of the short laser pulse relative to bunch:

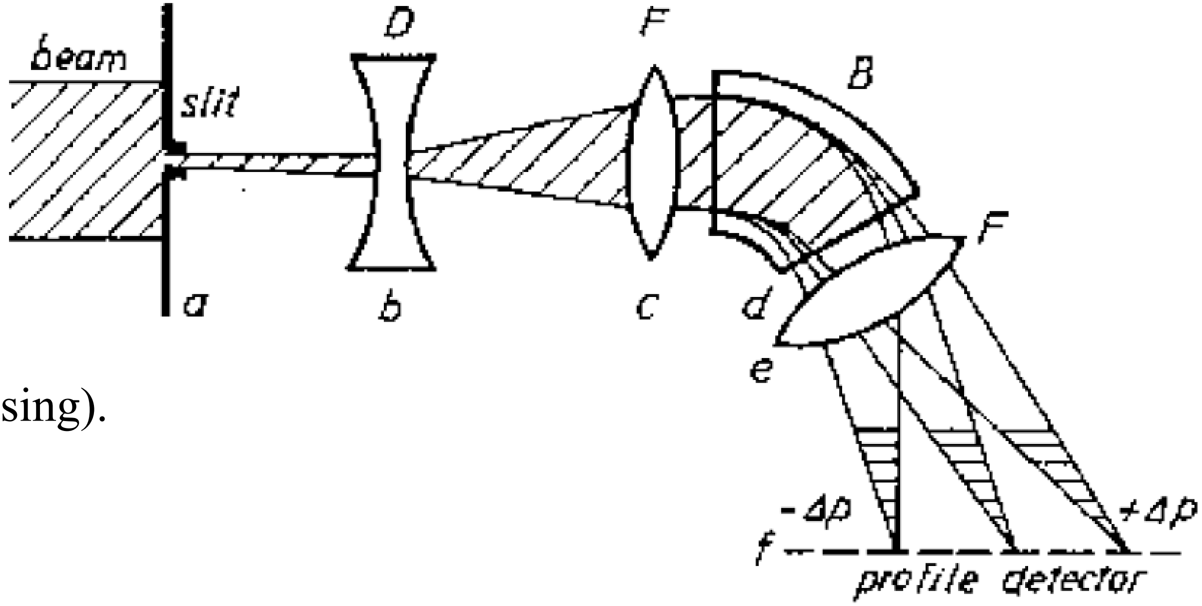


Results at FLASH, Hamburg, see B. Steffen et al., FEL Conf. Stanford, p. 549, 2005.

Measurement of Energy Spread by magnetic Spectrometer

The momentum $\delta = \Delta p/p$ or energy spread $\Delta W/W$ can be determined with a magnetic spectrometer:
→ Via dispersion, the momentum is shifted to a spatial distance.

The right optics has
to be chosen
to separate the transverse and
longitudinal parameters
(transverse point-to-point focusing).



Summary of longitudinal Measurements

Longitudinal ↔ transverse correspondences:

- position relative to rf ↔ transverse center-of-mass
- bunch structure in time ↔ transverse profile in space
- momentum or energy spread ↔ transverse divergence.

Determination uses:

Broadband pick-ups:
- position relative to rf, mean energy
- emittance at transfer lines or synchrotron via tomography
  assumption: bunches longer than pick-up.

Particle detectors:
- TOF or secondary e^- from wire
  → for non-relativistic proton beams
  reason: ***E***-field does not reflect bunch shape.

Streak cameras:
- time resolved monitoring of synchrotron radiation
  → for relativistic e^--beams, $t_{bunch} < 1$ns
  reason: too short bunches for rf electronics.

Laser scanning:
- Electro-optical modulation of short laser pulse
  → highest time resolution.