Schottky noise analysis

A beam is composed by a finite number of particles.
Schottky noise is based on the statistical fluctuations of this finite charge carriers.

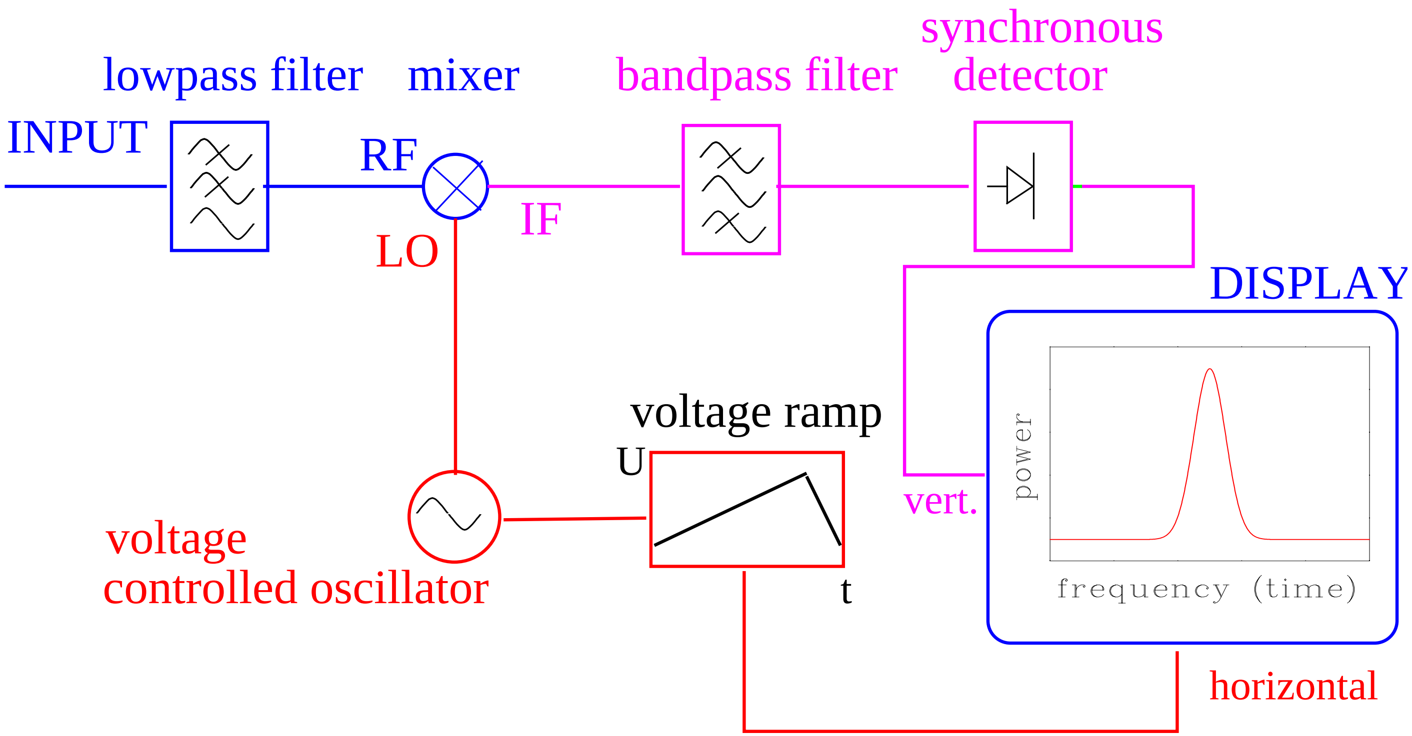
Schottky noise is treated in the frequency domain:
$\longrightarrow$ The sensitive device of a *spectrum analyzer* is used.
Modern applications: Fourier transformation of time domain data.

It is applicated at proton or heavy ion synchrotrons for:

- Measurement of momentum distribution $\Delta p/p$
  $\longrightarrow$ *longitudinal* Schottky
- Measurement of tune Q, chromaticity ξ and transverse emittance ϵ
  $\longrightarrow$ *transverse* Schottky

Spectrum analyzer

A spectrum analyzer is used to determine the amplitude of a frequency component of a time varying signal $\longrightarrow$ analog equivalent to a digital Fourier transformation.



- input signal is lowpass filtered: f_{cut} up to 3 GHz
- mixing with a scanned voltage controlled oscillator (typically 3 to 6 GHz)
- Schottky scan: typical span 10 to 100 kHz around 10 MHz, sweep time 0.1 to 1 s
- difference frequency is bandpass filtered (ty. 100 Hz) and rectified
- amplitude as a function of frequency is displayed

Longitudinal Schottky signals for a single particle

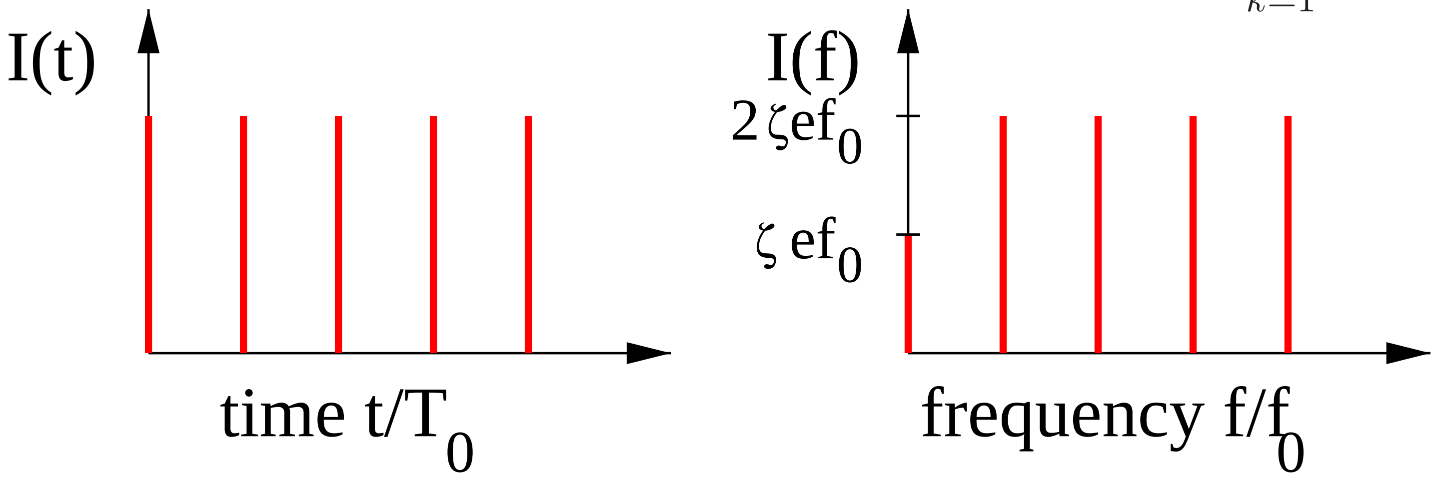
Consider one particle with charge ζe, rev. time T_0 and period $f_0 = 1/T_0$:

$$I(t) = \zeta e f_0 \sum_{h=1}^{\infty} \delta(t - hT_0) = \zeta e \sum_{h=-\infty}^{\infty} e^{2\pi i h f_0 t}$$

A real detector measures only positive frequencies:

$$I(t) = \zeta e f_0 + 2\zeta e f_0 \sum_{h=1}^{\infty} \cos(2\pi h f_0 t)$$

$\Longrightarrow$ Fourier transformation $\quad I(f) = \zeta e f_0 \cdot \delta(0) + 2\zeta e f_0 \sum_{k=1}^{\infty} \delta(f - k f_0)$



Longitudinal Schottky signals for a coasting beam

For N particles randomly distributed (de-bunched) along the synchrotron at angle θ_n the current is

$$I(t) = \sum_{n=1}^{N} \zeta e f_0 \cos\theta_n + 2\zeta e f_0 \sum_{n=1}^{N}\sum_{h=1}^{\infty} \cos\left(2\pi h f_n t + h\theta_n\right)$$

The power density per band at rev. harmonics h is the *rms* value $\langle I^2 \rangle$. Only the dc-part remains, all other cancel due to the averaging:

$$\begin{aligned}\langle I^2 \rangle &= \left(2\zeta e f_0 \sum_{n=1}^{N} \cos h\theta_n\right)^2 = \left(2\zeta e f_0\right)^2 \cdot \left(\cos h\theta_1 + \ldots + \cos h\theta_N\right)^2 \\ &= \left(2\zeta e f_0\right)^2 \cdot N/2 = I_{rms}^2\end{aligned}$$

Power per band determined by a pick-up with transfer impedance Z_t:

$$P(f) = Z_t \cdot I_{rms}^2/\Delta f = Z_t \cdot \left(\zeta e f_0\right)^2 \cdot 2N/\Delta f.$$

Fulfilled condition: $t_{scan} \gg T_0$, more precisely: resolution BW $\Delta f_{res} \ll f_0 = 1/T_0$

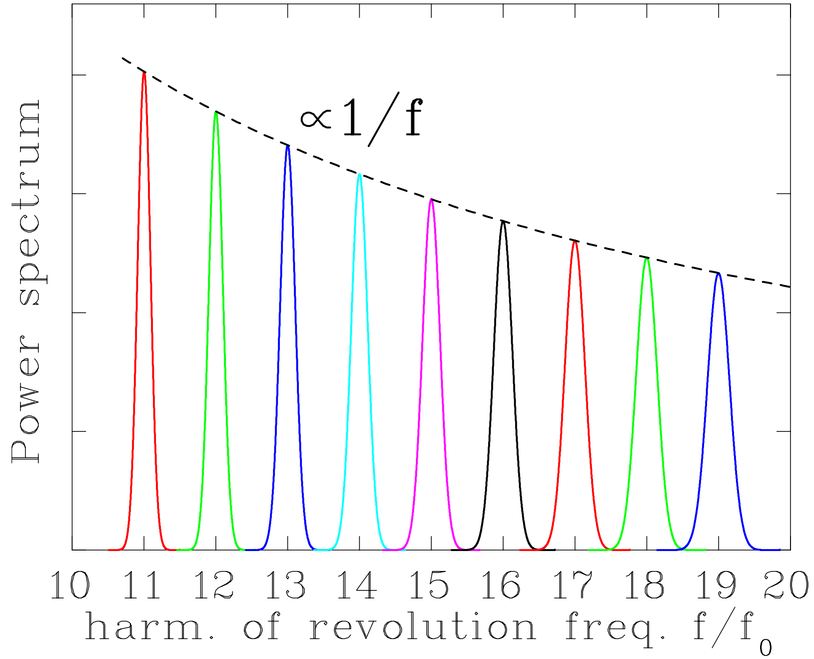
General longitudinal Schottky spectrum

The power per band is: $P(f) = Z_t \cdot I^2_{rms}/\Delta f = Z_t \cdot (\zeta e f_0)^2 \cdot 2N/\Delta f$.

Measured quantity:
Spread in rev. freq. $\Delta f_h/f_h$
$\Longrightarrow$ *momentum distribution*:

$$\frac{\Delta p}{p} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{f_h} = -\frac{1}{\eta} \cdot \frac{\Delta f_h}{h f_0}$$

For higher harmonics, Δf increases
$\Rightarrow$ Schottky peaks become broader.
Simulation using $\Delta p/p = 1$ % and frequency dispersion (art. low)
$\eta = 1/\gamma^2_{tr} - 1/\gamma^2 = -0.005$



Overlapping for $h > |\eta|/2 \cdot p/\Delta p$

Practical choice of the harmonics h:

Large values: Broader peaks $\longleftrightarrow$ higher resolution

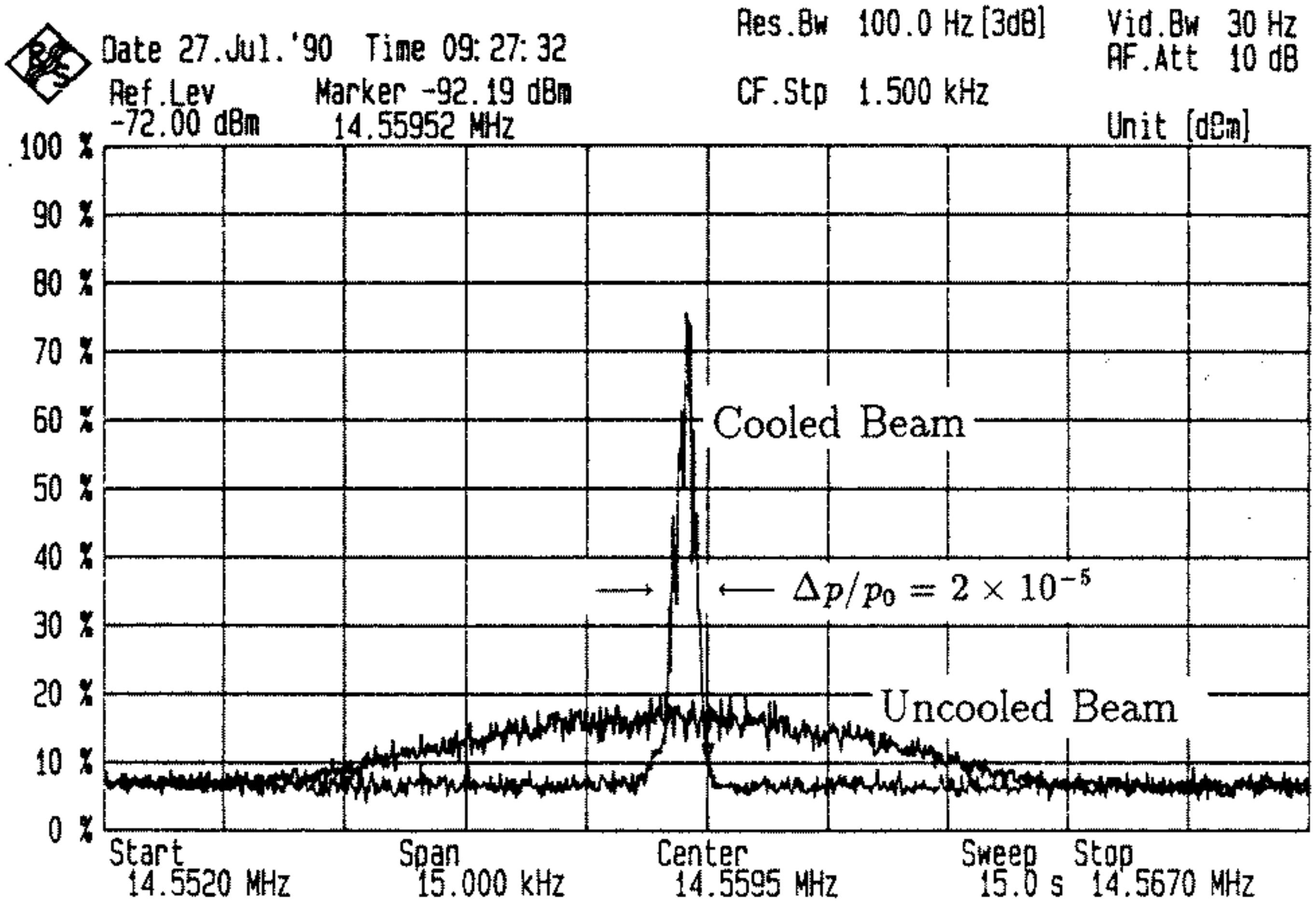
Low values: Better signal-to-noise due to $U_{noise} \propto \sqrt{\Delta f}$

$\Longrightarrow$ harmonics $10 < h < 30$ used ($f_0 \sim 1$ MHz at smaller heavy ion, $\overline{p}$ synchrotrons).

Results of longitudinal Schottky scan

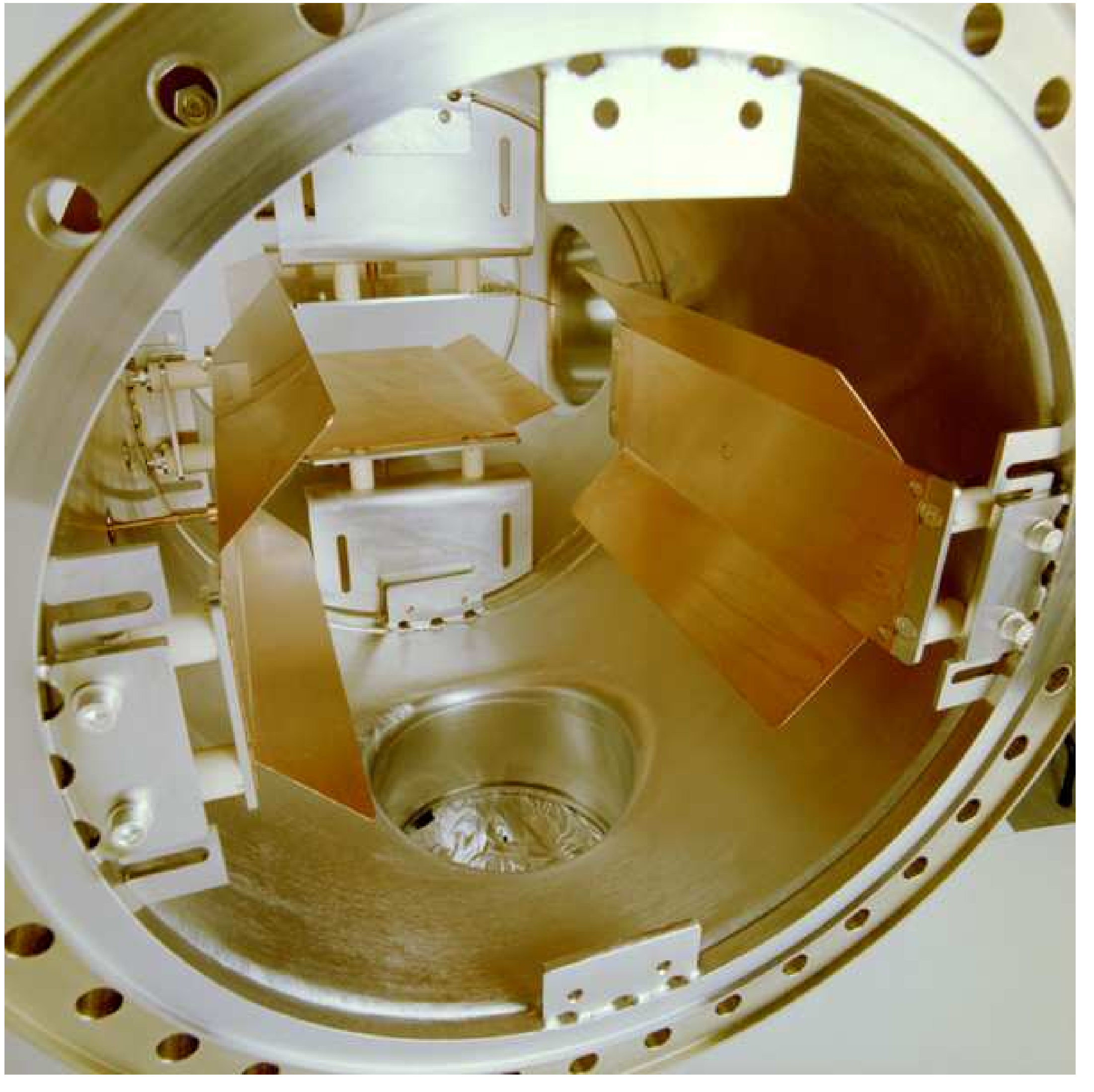
Control of longitudinal electron or stochastic cooling.

Example: Electron cooling of 300 MeV/u Ar^{18+} at GSI storage ring for harmonics 10 $\rightarrow$ decrease of the $\Delta p/p$ from $1 \cdot 10^{-3}$ to $2 \cdot 10^{-5}$:



Schottky pick-up at GSI synchrotron

In general it does not differ from a BPM; here 250 mm plate distance.



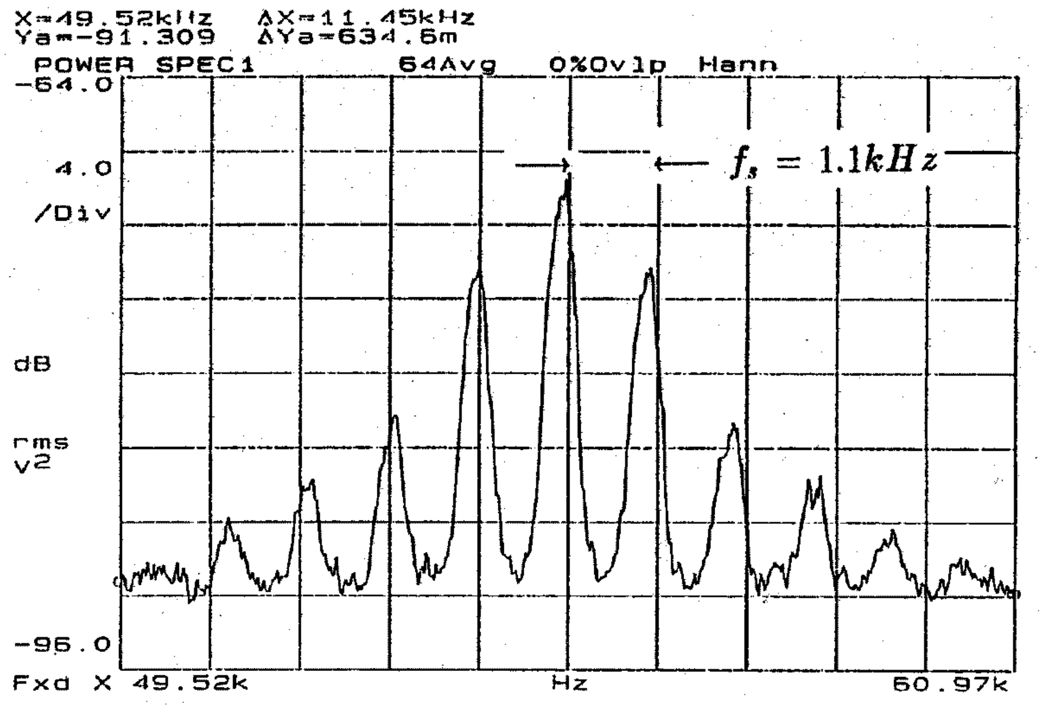
Longitudinal Schottky signals for a *bunched* beam

The statistical fluctuations are modulated by the synchrotron frequency f_s for each particle: $\tau_n = \hat{\tau}_n \sin(2\pi f_s t + \varphi_n)$

Modulation i.e. splitting of the long. peak according to:

$$I_h(t) = 2\zeta e f_0 \cdot \mathrm{Re}\left[\sum_{n=1}^{N} \sum_{p=-\infty}^{\infty} J_p(h\omega_n \hat{\tau}_n) \exp\left\{i\left(h\omega_n t + h\theta_n + p\omega_s t + p\varphi_n\right)\right\}\right]$$

The distance of peaks:
→ synchrotron frequency
$f_s \propto \sqrt{U_{rf}}$



Transverse Schottky signals for a *coasting* beam

The difference signal of opposite pick-ups are used.

The statistical fluctuations are modulated by the betatron oscillations:

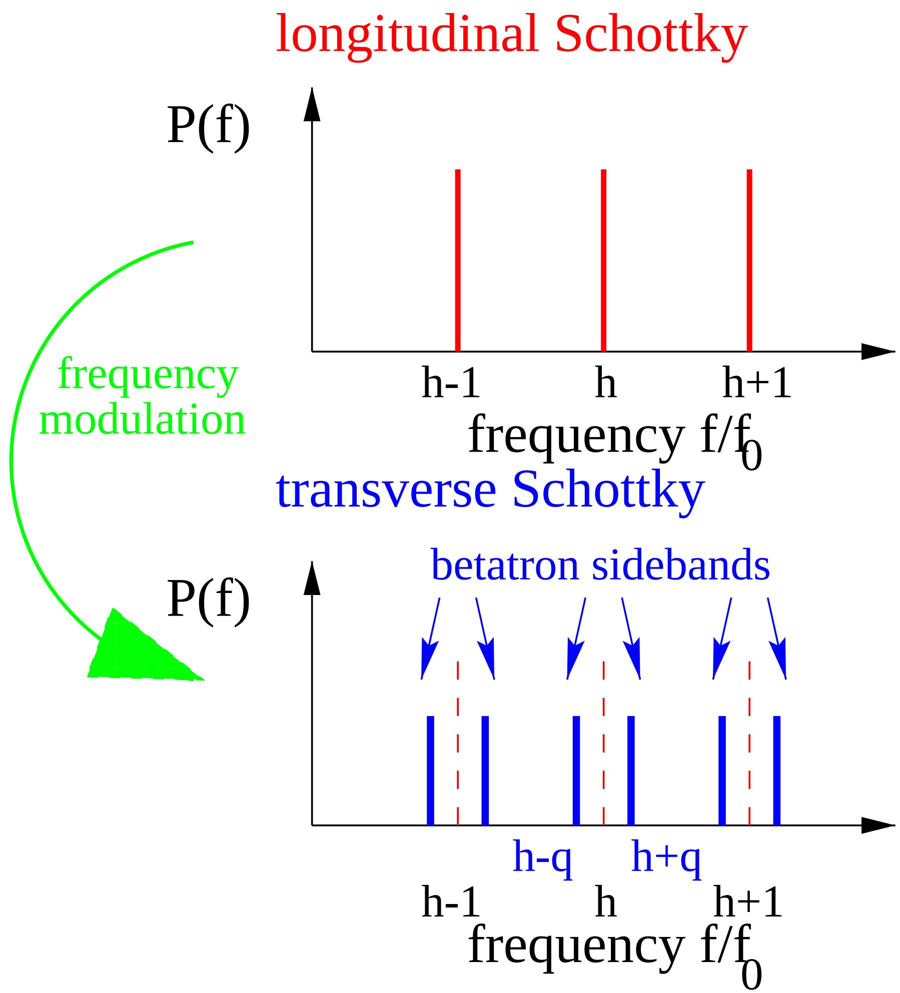
$$x_n(t) = A_n \cos(2\pi q f_0 t + \mu_n)$$

The dipole moment $d_n(t)$ for *one* particle is longitudinal noise $\propto \cos(2\pi h f_0 t)$

$\times$ betatron mod. $\propto \cos(2\pi q f_0 t)$

$\Longleftrightarrow$ frequency modulation

yielding:



$$d_n(t) = \zeta e f_0 \cdot A_n + \zeta e f_0 \cdot A_n \sum_{h=1}^{\infty} \cos\left[2\pi(h-q) f_0 t\right] \cdot \cos\left[2\pi(h+q) f_0 t\right]$$

Results for transverse Schottky scan → cooling

For N randomly distributed particles the rms power per band of transverse Schottky is:

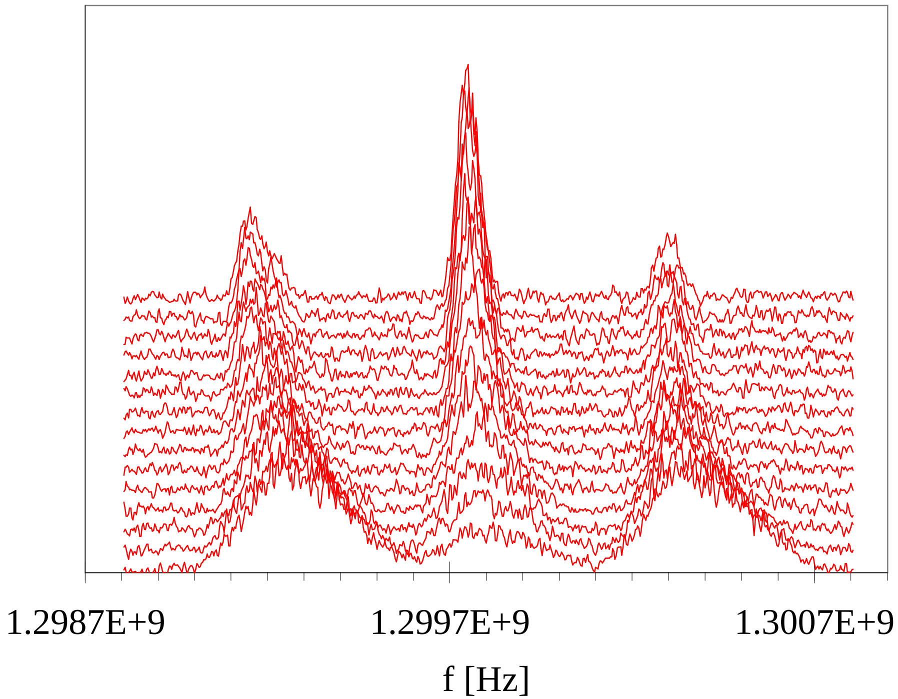
$$
\begin{aligned}
d_{rms}^2 &= \langle d^2 \rangle \\
&= (\zeta e f_0)^2 \cdot A_{rms}^2 \cdot N/2 \\
&= (\zeta e f_0)^2 \cdot \epsilon_x \cdot \beta(s) \cdot N/2
\end{aligned}
$$

$\longrightarrow d_{rms}^2 \propto \epsilon_x$ (trans. emit.)
(i.e. integral of sideband)

Transverse Schottky:
$\sim 1/100$ smaller than longitudinal.

Absolute value of emittance:
Calibration necessary due to unknown transverse transfer impedance $Z_\perp(\omega)$

Transverse Schottky spectra recorded every 80 ms during stochastic cooling
$\longrightarrow$ decrease of sidebands $\Leftrightarrow$ trans. cooling



Transverse Schottky scan → tune

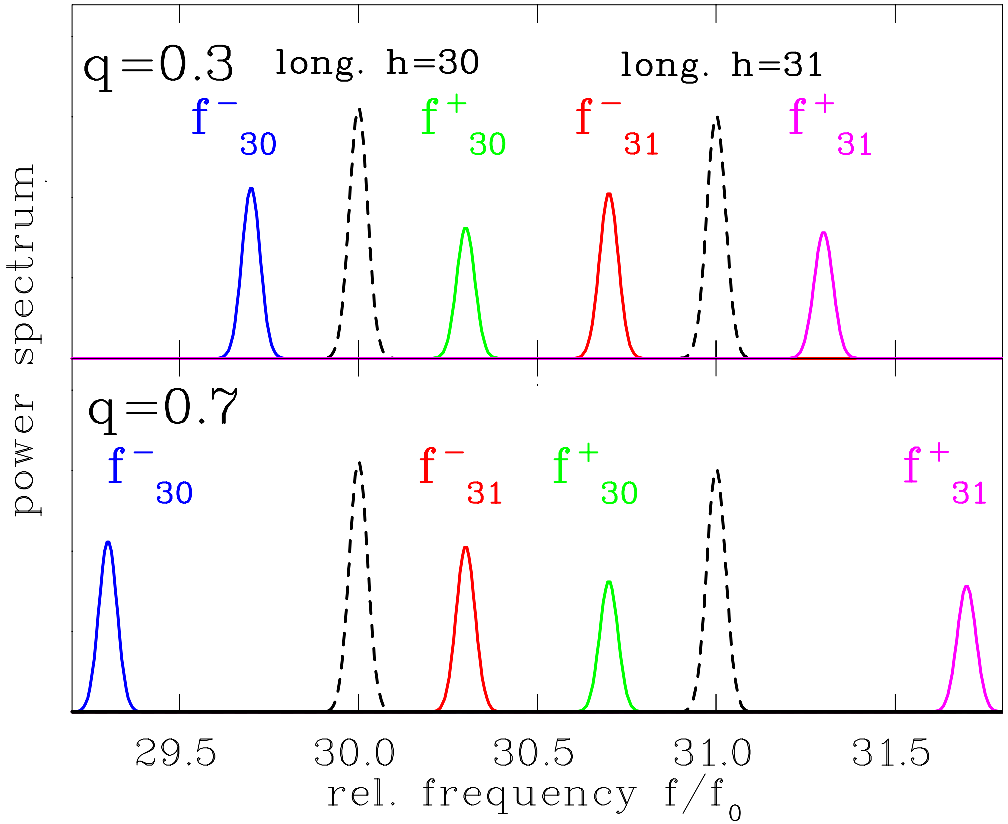
The position of the sideband are given by the non-integer tune q.

Frequency modulation with

$\cos\left[2\pi(h \pm q) f_0 t\right] \Rightarrow$

$$q = h \cdot \frac{f_h^+ - f_h^-}{f_h^+ + f_h^-}$$

⇒ direct tune measurement



$h = 30$, 31: $q = 0.3$ sideband close, $q = 0.7$ interchanged.
Parameter: $f_0 = 1$ MHz, $\Delta p/p = 2 \cdot 10^{-3}$, $\eta = -1$, $\xi = -1$,
$Q = 4.3$ or 4.7.

Transverse Schottky scan → chromaticity

The width of the sideband $\Delta f_h^{\pm}$ is related to the chromaticity ξ.

Frequency modulation with
$\cos\left[2\pi(h \pm q) f_0 t\right]$.

Position of sidebands:
$f_h^{\pm} = (h \pm q) f_0$

and product role of derivative ⇒

$$\Delta f_h^{\pm} = \Delta f_0 (h \pm q) \pm \Delta q f_0$$

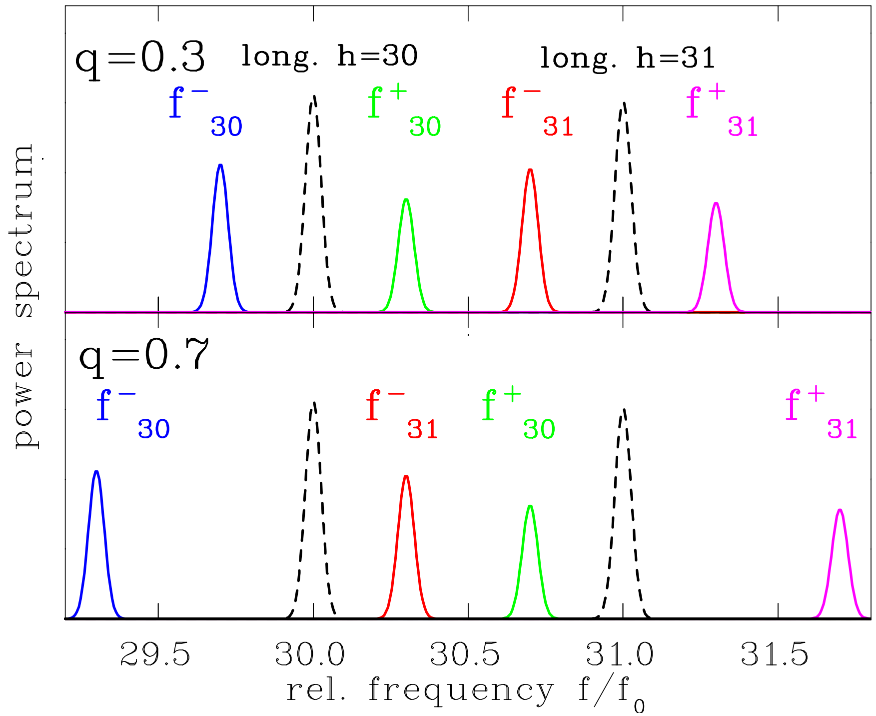
With $\Delta f / f = -\eta \cdot \Delta p / p$

and $\Delta Q / Q = \Delta q / q = \xi \cdot \Delta p / p$

$$\Delta f_h^{-} = \eta \frac{\Delta p}{p} \cdot f_0 \left(h - q + \frac{\xi}{\eta} Q \right)$$

$$\Delta f_h^{+} = \eta \frac{\Delta p}{p} \cdot f_0 \left(h + q - \frac{\xi}{\eta} Q \right)$$

The chromaticity ξ is determined without beam excitation.



Parameter: $f_0 = 1$ MHz, $\Delta p/p = 2 \cdot 10^{-3}$, $\eta = 1$, $\xi = -1$, $Q = 4.3$ or 4.7.
Care: The width of the sidebands differs!

Some remarks to Schottky analysis

- Transverse Schottky for a bunched beam → sidebands are modulated due to synchrotron oscillations → complex spectrum.
- The longitudinal spectrum can significantly deformed by observing cooled beams, preventing the interpretation of the width as the momentum spread. For cold and sufficient dense beams, the signal shows a splitting related to plasma waves.
- Signal enhancement with external resonator is possible for low currents like anti-protons.